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STATEMENT AND EXPOSITION

OF

CERTAIN HARMONIES

OF

THE SOLAR SYSTEM.

BY

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ADVERTISEMENT.

The principal part of the following Memoir on Certain Harmonies of the Solar System was read before the American National Academy of Sciences, at its meeting in April, 1873, and some additional portions of the same, at the meeting in April, 1874.

In accordance with usage in such cases the whole is now presented to the public through the Smithsonian Contributions to Knowledge.

JOSEPH HENRY,
Secretary S. I.

Note by the Author.—After reading the whole memoir, a synopsis of the principal relations may be obtained by a reperusal and comparison of the Tables (B) to (F) inclusive, with their explanations; and, especially, the Summation of Consistencies at the end.

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N.B. The references are to the articles, not to the pages.

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CERTAIN HARMONIES OF THE SOLAR SYSTEM.

SECTION I.

INTRODUCTORY.

(1) KEPLER's 3d Law is ordinarily expressed by saying that the squares of the periodic times of the several planets of the solar system are to one another, respectively, as the cubes of their distances from the sun. The same law includes also the periodic comets, and it is, in like manner, applicable to the satellite systems.

But from this we do not learn that any laws are to be found determining the ratios of the distances themselves.

It will be one main object of the present discussion to show that such laws exist, and precisely what they are—generality and precision being characteristics of every law of nature.[1]

(2) Approximations to the laws in question have, from time to time, been exhibited, by the author of this paper, to the American Association for the Advancement of Science, at several of their meetings, beginning with that at New Haven, in 1850, and more especially, also, that at Montreal, in 1857; Baltimore, in 1858; and Springfield, Massachusetts, in 1859; but it is only within the past few months, or even almost up to this present time (July, 1874), that the entire form and consistency of the results hereinafter exhibited have been quite fully made out.

(3) All that is to be stated will, it is conceived, be the more readily intelligible by proceeding, as occasion may seem to require, *inductively*, and consequently following, to some extent, the order of discovery.

Antecedently even to this, however, it seemed to be desirable to discuss anew the expressed values of the distances in question, and this, in view of the fact, that Kepler's 3d Law is itself slightly modified by the consideration due to the *masses* of the revolving bodies.

Thus if M represent the mass of the sun, and m, m' the respective masses of any two planets, while a, a' represent their mean distances from the sun, and T, T' represent their periodic times, we have

$$\left.\begin{array}{l} \left(\frac{T'}{T}\right)^2 = \left(\frac{a'}{a}\right)^3 \times \frac{M+m}{M+m'}, \text{ or} \\ \left(\frac{T'}{T}\right)^2 \times \frac{M+m'}{M+m} = \left(\frac{a'}{a}\right)^3 \end{array}\right\} \ldots (1);$$

[1] The so-called Law of Bode or of Titius, it need scarcely be said, fails in both these respects.

When m and m' are mere particles of matter *Eqs.* (1) are both reduced to

$$\left(\frac{T'}{T}\right)^2 = \left(\frac{a'}{a}\right)^3 \ldots \quad (1)'.$$

It may be convenient to regard, once for all, a, m, and T, in so far as they appear, as being special for the earth, while a', m', and T' respectively represent like quantities in the instance of any other planet.

Now T' and T having both been well ascertained, and being themselves constant, the same is true of their ratio, which involves also the *constant value* of $\left(\frac{T'}{T}\right)^2$; and hence it follows that, to preserve *Eq.* (1)', we must have the value of $\left(\frac{a'}{a}\right)^3$ also *constant*, and this, although the accepted value of a, the earth's mean distance from the sun, which is the unit of measurement, may itself require correction in comparison with other standards. If *it* then be diminished, every other mean distance a', as it is represented in *Eq.* (1)', will be found to be diminished in the same ratio; and thus, while the numbers representing them remain *unchanged*, "all the distances have to be reckoned on a new scale."[1]

Next, as respects the modifying factor $\frac{M + m'}{M + m}$, in the second of *Eqs.* (1). As it is moreover true, that M itself varies directly as a^3; if a^3 be *diminished*, M will be *diminished* in the same ratio, and *the like* will be true of m' represented, as usual, in terms of M as the measuring unit; so that all such masses will be represented by the same numbers as before, but all, as in the case of the distances, "reckoned on a new scale," while *the mass of the earth* will, in this comparison, be *increased*, as that will vary *inversely* as a^3.

Now the more recent determination of the solar parallax requiring that the actual value of a should be diminished, it became requisite for the accurate determination of the values of the mean distances of such other planets as have ascertained and appreciable masses, that those values, as already intimated, should be rediscussed.

This has been done with the aid of logarithms computed to ten decimal places of figures; and the results, to the seventh decimal place inclusive, are exhibited in Table (A), in which withal, in their appropriate column, are also the values of the masses made use of, with indications of the authorities to which they are referable.

The densities which besides are exhibited in Table (A), will be found to vary more or less from those hitherto ordinarily accepted. This is due to the increase in the relative mass of the earth, and also to the more accurate determination of the masses of the planets.

The arrangement of the series of planets begins with the most distant, as that will be found to be the more convenient for the application of these data to the special purposes of the whole investigation.

[1] Sir J. Herschel's *Outlines of Astronomy*, 11th edition (357 *c*.)

The results given are those which are respectively consistent with two values of the solar parallax; viz., Prof. Newcomb's value $\pi = 8''.848$,[1] and that which some prefer, $\pi = 8''.78$.

TABLE (A).

A Synoptic Table of some of the Elements of the Planetary System.

	Names.	Periodic Times.	Masses ($\pi = 8''.848$.)	Masses ($\pi = 8''.78$).	Mean Distances. ($\pi = 8''.848$.)	Mean Distances ($\pi = 8''.78$).	Densities ($\pi = 8''.848$).	Densities $\pi = 8''.78$).
♆	Neptune,	60186ᵈ.6385	$\frac{1}{19700}$	$\frac{1}{19700}$	30.0567298 —	30.0567339 —	0.142 —	0.145 +
♅	Uranus,	30688 50	$\frac{1}{22000}$	$\frac{1}{22000}$	19.1833617 +	19.1833622 —	0.182 —	0.186
♄	Saturn,	10759.2198174	$\frac{1}{3501.600}$	$\frac{1}{3501.600}$	9.5388544 —	9.5388546 —	0.119 —	0.122 —
♃	Jupiter,	4332.5848212	$\frac{1}{1047.879}$	$\frac{1}{1047.879}$	5.2028004 —	5.2028005 —	0.240 —	0.245
♂	Mars,	686.9796458	$\frac{1}{3200900}$	$\frac{1}{3200900}$	1.5236913	1.5236913 +	0.585 +	0.599 +
⊕	Earth,	365.2563582	$\frac{1}{322800}$	$\frac{1}{330358}$	1.0000000	1.0000000	1 000	1.000
♀	Venus,	224.7007869	$\frac{1}{408134}$	$\frac{1}{408134}$	0.7233322 —	0.7233322 —	0.809 +	0.828 +
☿	Mercury,	87.9692580	$\frac{1}{4865751}$	$\frac{1}{4865751}$	0.3870987 —	0.3870987 —	1.122 —	1.148 +
☉	Sun,	....	1	1	....	....	0.250 +	0.256 +

REMARKS.—The authorities for the *Periodic Times* are:—

Uranus. From Prof. NEWCOMB's Tables of Uranus.

Earth. The sidereal year of HANSEN and OLUFSEN, as quoted by Prof. WATSON. *Theor. Astronomy*, Table XXI.

The other periodic times are those usually accepted.

For the *Masses* we have—

Neptune. The Pulkova deduction, furnished by Prof. NEWCOMB.

Uranus. From Prof. NEWCOMB's Tables of Uranus.

Saturn. BESSEL, *Comptes Rendus*, 1841.

Jupiter. BESSEL, Die Masse des Jupiter, p. 64. [Its great accuracy is confirmed by Prof MÖLLER's deduction from the perturbations of *Faye's Comet*, and by the recent investigations by Dr. KRUEGER, of the perturbations of *Themis*, *Ast. Nachrichten*, No. 1941.]

Mars. HANSEN and OLUFSEN's mass, as quoted by Prof. HILL. *Tables of Venus*, p. 2.

Earth. Prof. NEWCOMB's *Investigation of the Distance of the Sun*, etc., § 11 (with $\pi = 8''.848$). With $\pi = 8''.78$, the mass was deduced, with a change of value proportioned to π^3.

Venus. Prof. HILL, *Tables of Venus*, p. 2.

Mercury. ENCKE, *Astronomische Nachrichten*, No. 443.

The columns of densities have been computed by the aid of the other data. If we admit for *Venus* the mass $\frac{1}{427240}$, to which some indications point (*Hill's Tables*, p. 2), then the density of that planet with the value of the solar parallax $= 8''.848$, will be represented by 0.773, or for the value of $\pi = 8''.78$, the representative density will be 0.791 +. The only change in the value of the mean distance of Venus will then be that the last decimal figure (with $\pi = 8''.848$) will read 1 + instead of 2 —.

[1] Smithsonian Contributions to Knowledge—Investigation of the Distance of the Sun, etc., § 10.

SECTION II.

ON THE LAWS OF ARRANGEMENT OF THE DISTANCES, BOTH OF PLANETS AND THEIR SATELLITES, FROM THEIR RESPECTIVE CENTRES OF ATTRACTION.

(4) The object of this section is to indicate distinctly the ratios which prevail among the planetary and satellite distances from their respective centres, and also the *laws* which include the same; *without the introduction in this same connexion of any physical hypothesis on which those laws seem to be founded, or of which they are the exponents.*

The hypothesis which seems to reconcile and explain those laws, *as well as a number of other phenomena,* will be considered in a subsequent section.

(5) The first correspondence and arrangement of ratios that will be noticed, may be thus stated: Beginning with the mean distance of Neptune as found in Table (A) in (3), if of this we take $\frac{5}{9}$, and of that fractional product, again, $\frac{5}{9}$, etc., etc.; then, among *the terms in the geometrical progression thus developed,* in addition to that pertaining to Neptune, we shall find those which respectively, in their order, exhibit close approximations to the mean distances of the two great planets Saturn and Jupiter; another having an appropriate position among the asteroids;[1] with, again, others which respectively exhibit close approximations to the mean distance of Mars, and that of Mercury *in aphelion;* all which can be distinctly traced in the following tabular arrangement, in which the approximations are carried to the third place of decimals inclusive; though the computations were extended to the fifth place. In the third column, it will be remembered, every *term* after the first, is $\frac{5}{9}$ *of that immediately preceding;* so that the ratio of every one to *its next succeeding term* will be that of 9 to 5 = to $\frac{9}{5} = \frac{18}{10} = \frac{1.8}{1} = 1.8$; a statement which, in certain comparisons, will be found to be more convenient than the other.

In this arrangement the column under the title of *Law* exhibits the results in accordance with the (approximate) law of succession of the terms as now explained; in comparison, respectively, with the recorded distances found in the column of *Fact; the terms in the column of Law forming a series in geometrical progression, the ratio being* 1.8.

1st *Approximate Arrangement.*

Names and Symbols.		Law.	Fact.	Difference. L.—F.
♆	Neptune,	30.05733	30.05733	0 000
♅	{ Uranus,	....	{ 19.183+	....
(U)	{ *Limit* (U),	16.698+	{	....
(♅′)	{	....	{ (missing)	
♄	Saturn,	9.277—	9.539—	—0.262
♃	Jupiter,	5.154—	5.203—	—0.049
(A)	*Limit* (A),	2.863+	(to be supplied)	....
♂	Mars,	1.591—	1.524	+0.067
⊕	{ Earth,	....	{ 1.000	....
(⊕♀)	{ *Limit* (⊕♀),	0.884—	{	....
♀	{ Venus,	....	{ 0.723+	....
Aph. ☿	Mercury, in Aphelion,	0.491—	0.467—	+0.024

[1] Of which more hereafter.

(6) An inspection of what is here exhibited will at once reveal the fact that the Earth and Venus seem to have characteristics of *half-planets;* the *one* term, 0.884 (in the series), pertaining to them, being indicative of a distance between those of the two planets at which their masses should be united; and which is designated as *limit* (⊕ ♀).

[To avoid circumlocution, such an arrangement as this, will be termed *a half-planetary arrangement*, and the planets subject to it, be, at times, designated as *half-planets;* those situated, as Uranus and the Earth are, *without* the intervening *limit*, being styled *exterior* half-planets; while those, like Venus, *within the limit*, are specially designated as being *interior* half-planets; Uranus being regarded as an *exterior* half-planet as well as the Earth. For the ratio of the mean distance of Neptune to that of Uranus is very *nearly the same as that of Mars to the Earth's;* viz., a very little greater than the ratio of $1\frac{1}{2}$ to 1. And so[1] the *limit* (U) in the *progression* is very nearly *the same fraction* of the term for *Uranus* in the column of Fact, that the *limit* (⊕ ♀) is of the *Earth's distance;* viz. very nearly $\frac{9}{10}$, in both cases.]

(7) Uranus, then, like the earth, has the characteristics of an *exterior half-planet;*[2] though there is no other half-planet (analogous to Venus) apparent between *limit* (U) and Saturn. But *the region* of the system where the appropriate term for *such* a half-planet should be found has been marked in the tabular arrangement, and its symbol (⛢*i*) shows that *it* would belong to *a half-planet interior* to Uranus; such as Venus is in the region interior to the Earth's place.

(8) Now the ratios for the mean distances from the Sun of the *exterior* half-planet terms, are as follows:—

$$\left.\begin{aligned}\frac{\textit{Neptune}}{\textit{Uranus}} &= 1.56681\\ \frac{\textit{Mars}}{\textit{Earth}} &= 1.52369\\ \frac{\textit{Mercury in aphelion}}{\textit{Mercury in perihelion}} &= 1.51768\end{aligned}\right\}\ \text{Mean} = 1.53606;$$

while it is also true, with respect to the ratio for other than half-planet distances [which $= \frac{9}{5}$ or $\frac{1\cdot 8}{1}$ very nearly], that

$$(1.8)^{\frac{3}{4}} = 1.55401,$$

agreeing very nearly with the preceding; so that, r being the ratio for other than half-planets, the ratio for the *exterior* half-planets is $r^{\frac{3}{4}}$.

Also, as again respects mean distances from the Sun,

$$\frac{\textit{Earth}}{\textit{Venus}} = 1.38249.$$

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[1] *Having all the while in view* the table of the first Approximate Arrangement under discussion.

[2] This was not discerned until just before the Meeting of the American Association for the Advancement of Science, in Baltimore, in 1858. It is just the non-perception of a *half-planet* relationship, that has seriously troubled most of the investigations into the arrangements, etc., of the planetary system, whether purely speculative or otherwise.

But r being still $= 1.8$, *the square root of* r, or

$$r^{\frac{1}{2}} = 1.34161,$$

so that, r being still the leading ratio, the ratio for the *interior* half-planet Venus, is $r^{\frac{1}{2}}$; and this planet furnishes the only *existing* example of its kind in the planetary system. Another will appear in the system of Saturn.

The relations thus ascertained may be symbolized as follows; the dependence of a following term on that from which it is *derived* being indicated by a brace connecting the two, and the power of r involved marked outside of the brace: as, for example, we have

Mars — Planet, ½ *planet* (brace) $r^{\frac{3}{4}}$ — *Mars*, *Earth*/*Venus* Limit (⊕♀) (brace) r

Earth . . . . $r^{\frac{1}{2}}$ { ½ *planet* . . . . ½ *planet* } Pl. limit . . { *Earth* / *Venus* } Limit (⊕♀)

Venus

Planetary limit. . . . . Aphelion of Mercury. — Limit (⊕♀), Aphelion of Mercury (brace) r

(9) This being kept in view, it will be apparent from what precedes, that the rules now established for the derivation of all the distances in the planetary arrangement subsequent to the first, are as follows:—

[Leading ratio r being $= 1.8$ very nearly]

Rule 1*st.*—When the *term in question* in the series of planetary distances is *other* than that pertaining to a *half-planet*, the value of that term may be obtained by dividing the value of the term immediately preceding by the leading ratio.

Examples.—Thus, as indicated by the symbols,

$$\frac{\textit{Saturn term}}{r} = \textit{Mean distance of Jupiter}$$

$$\frac{\textit{Mars term}}{r} = \textit{Limit } (\oplus ♀); \text{ and}$$

$$\frac{(\oplus ♀)}{r} = \textit{Aphelion distance of Mercury.}$$

[This (incidentally it may be) includes the term for *Mercury*,[1] with the variety, that the term which immediately precedes (and which is to be employed in *that* computation) is the term pertaining to the *half-planet Venus;* though Mercury itself is not a half-planet, but even has characteristics approaching to those of a *double-planet.*]

Rule 2*d.*—The value of *any term* in the series of *exterior* half-planets may be obtained by dividing the value of the term immediately preceding *that* in the planetary arrangements, by $r^{\frac{3}{4}}$.

[The *Examples* are: The respective mean distances of Uranus and the Earth, and the *perihelion* distance of Mercury. Thus,

$$\frac{\textit{Mars term}}{r^{\frac{3}{4}}} = \textit{Earth term.}]$$

[1] Incidentally, it *may* be; for Mercury's mean distance has other relations; as will appear in Section III.

Rule 3d. — The value of *any term* in the series of *interior* half-planets may be obtained by dividing the value of the term of the planetary arrangement immediately preceding *that*, by $r^{\frac{1}{2}}$.

[*Examples* are: The mean distance of Venus, and that due to the *missing interior half-planet*, next in the arrangement to the *exterior* half-planet Uranus. Thus

$$\frac{\textit{Earth term}}{r^{\frac{1}{2}}} = \textit{Venus term.}]$$

With D', or D'', or D''', as the case may be, for the value of the distance in question, and D that to which that value is referred, we have

For *Case* under *Rule First*,

$$D' = \frac{D}{r}; \text{ whence, withal, } r = \frac{D}{D'} \quad . \; . \; . \; . \; . \; (a)$$

$$\left[\text{For Mercury, } D' = \frac{(d)}{r}\right]^1$$

For *Case* under *Rule Second*,

$$D'' = \frac{D}{r^{\frac{3}{4}}}$$

For *Case* under *Rule Third*,

$$D''' = \frac{D}{r^{\frac{1}{2}}}$$

From these equations we also learn, that

$$\left.\begin{array}{l} \dfrac{D'}{D}, \text{ or } \dfrac{D'}{(d)}, \text{ each } = \dfrac{1}{r}, \\ \dfrac{D''}{D} = \dfrac{1}{r^{\frac{3}{4}}}, \text{ and} \\ \dfrac{D'''}{D} = \dfrac{1}{r^{\frac{1}{2}}} \end{array}\right\} \quad . \; . \; . \; . \; (\mathrm{P})$$

(10) These equations express the laws of apportionment of the planetary distances; which are these: —

Laws of Apportionment of the Planetary Distances.

[Value of $r = 1.8$, very nearly.]

LAW FIRST. For any term subsequent to the first, in the series of terms of planetary distances; and *other than* a half-planetary term:—

succeeding term : *prec. term* : : 1 : *leading ratio r.*

LAW SECOND. For an *exterior* half-planetary term:—

ext. half-planet. term : *prec. term* : : 1 : *¾ power of leading ratio r*, i. e. $r^{\frac{3}{4}}$.

LAW THIRD. For an *interior* half-planetary term.

int. half-planet. term : *prec. term* : : 1 : *square root of leading ratio r, or* $r^{\frac{1}{2}}$.

---

[1] (*d*) being the term pertaining to the *interior half-planet Venus.*

In the second approximate arrangement which follows, the dependence of the value of one term on that of another is indicated by *the brace* connecting them, and the power of r in question is also shown; the half-planetary terms have their names printed in italics; while Mercury's name (in view of the peculiarity of that planet) appears in capitals: other symbols, etc., as heretofore.

The leading ratio here accepted, after many trials of *it* and of other ratios, is 1.805.

Second Approximate Arrangement of the Planetary System. [*Value of Leading Ratio* 1.805].

Names and Symbols.				Law.	Fact.	Diff. L.—F.
♆		Neptune } $r^{\frac{1}{2}}$,	}	30.05673	30.05733	—0.001
⛢		*Uranus* }	r	19.30118	19.18336	+0.118
(U)	$r^{\frac{1}{2}}$ {	Limit (U),		16.65193	....	....
⛢ i		....	r	....	(Missing).	....
♄		Saturn,		9.22545	9.53885	—0.313
			r			
♃		Jupiter,		5.11105	5.20280	—0.092
			r			
(A)		Limit (A),		2.83161	....	....
			r			
♂		Mars } $r^{\frac{1}{2}}$		1.56876	1.52369	+0.045
⊕		*Earth* }	r	1.00739	1.00000	+0.007
(⊕♀)	$r^{\frac{1}{2}}$ {	Limit (⊕ ♀),		0.86912	....	....
♀		*Venus*,	r	0.74982	0.72333	+0.026
Aph. ☿	r {	Mercury in Aph. }		0.48151	0.46670	+0.015
☿		MERCURY } $r^{\frac{1}{2}}$		0.41543	0.38710	+0.028
Per. ☿		*Mercury in Per.*		0.30920	0.30750	+0.002

(11) The approximation of law to fact here shown, though in the main very close, yet exhibits some terms in which the discrepancy is a greater fraction of the whole than seems to be quite tolerable, in view of the accuracy of the other terms.

Then, too, the last column of the arrangement here shows a tendency in the difference of law from fact to be *negative* for the *first* part of the series of terms, *but positive afterwards;* as though the value of the leading ratio were in excess for the one portion, and thus had given the results in general too small; but the same value of the ratio having been too small in the case of the remaining terms, had consequently given results too large. All this makes it not improbable that the leading factor r, from first to last, should regularly increase, beginning *below* the mean value of 1.805, and ending *above* the same; the increase, however, in any event, being *very small.*

To ascertain whether this is so, it will be found advisable to institute a separate induction within the narrower limits of the region from Saturn to Mars inclusive, in which we possess three out of the four requisite terms;[1] the fourth (the asteroid term or limit (A)) to be accurately determined by the process here proposed, and its value *thus* obtained to be made *the criterion* for the comparison of its value as ascertained in the more extended series. In the several instances of the three planets here in question, there are withal no *half-planet* relations, and the fourth term being a *limit* in the regular series in which r enters, the half-planet relation does not pertain to *it;* so that the character of the leading factor r, as to variability or otherwise, is here to be sought for.

(12) Now the *existing* mean distances from the sun in this region, together with *the asteroid limit* (A), may be arranged as follows, viz.:—

	Dist. from Sun.[2]	Log. of Ratios.	Difference.
Saturn	9.53885 +		
		0.2632591 —	
Jupiter	5.20280		
		0.2655331 —	+ 0.002274
Limit (A)	(2.82296 —)		0.002274
		0.2678071 —	
Mars	1.52369 +		

The log. differences being equal, the ratios themselves increase *in geometrical progression.*

But if the arrangement be made with the ratios increasing *in arithmetical progression*, we shall have—

	Dist. from Sun.[2]	Ratios.	Difference.
Saturn	9.53885 +		
		1.83341	
Jupiter	5.20280		+ 0.00964
		1.84305	
Limit (A)	(2.82293 —)		0.00964
		1.85269	
Mars	1.52369 +		

Now we do not know enough of the nature of the case to decide which of these conditions ought to prevail, though the analogy of logarithms etc. would lead us to suppose that the ratios themselves should increase in arithmetical progression. But, happily, such a decision is of no moment practically; since the differences in question are so small, that the value of the limit (A) in the one case differs from that in the other only in the fifth decimal place.

So the value of the limit (A) = 2.82293–, which is that due to the increase of the ratio in *arithmetical progression*, will be accepted, and the same will be adopted; and then, as heretofore intimated, this value will be made the *criterion* for the comparison of the value as ascertained in the more extended series. This standard value, being withal a direct derivation from fact, in its own special region, will hereafter be inserted as *a limit* in the column of *Fact*, the figures being inclosed in a parenthesis.[3]

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[1] In the order of discovery, it was in *this region* that the approximation of the series of distances to a geometrical progression, with the ratio = 1.8 nearly, was first discerned.

[2] See Table (A), in (3).

[3] This value, 2.82293, is greater than the mean of the distances from the sun of 122 known asteroids, which is only 2.70282. But then about $\frac{7}{12}$ of that number are distances below the mean; leaving but $\frac{5}{12}$ above the same. So that it seems not unreasonable to suppose that were many more included, which mostly are now unknown—partly, it may be, because of their greater distance—the mean

(13) The *increment* of the leading ratio, or factor r, having been ascertained to be *real* for the region thus examined, an application of the rule which that implies was tried throughout the planetary system; and after an enormous number of such tentative processes, the following local values of r were found to give the most consistent results, the values of r, it will be seen, increasing withal in arithmetical progression.

Values of r in the Planetary System.

Region.	*Factor r.*	
Neptune to *limit* (U)	1.7770	
Limit (U) to Saturn	1.7908	0.0138
Saturn to Jupiter	1.8046	0.0138
Jupiter to *limit* (A)	1.8184	0.0138
Limit (A) to Mars	1.8322	0.0138
Mars to *limit* (⊕♀)	1.8460	0.0138
Limit (⊕♀) to the Aphelion of Mercury	1.8598	0.0138
Aphelion of Mercury to *limit* within	1.8736	0.0138

The mean of these is 1.8253; differing a little less than $\frac{1}{38}$th of itself from either extreme.

From these we have for the *exterior* half-planet intervals:—

Region.	*Factor* $r^{3/4}$.
Neptune to Uranus	1.5369 —
Mars to Earth	1.5710 —
Aphelion to Perihelion of Mercury	1.6014 +

For the *interior* half-planet intervals, we have:—

Region.	*Factor* $r^{1/2}$.
Uranus to ♅i	1.3356 +
Earth to Venus	1.3612 +

From the *interior* half-planet Venus to Mercury

$$r = 1.8632 +$$

Under these conditions the value of the ***half-planet limit*** **♅i, *i.e.*** interior to Uranus, may now be determined; and it will be found to be 14.64275.[1]

(14) The arrangement of the planetary system in accordance with all that has now been determined, is similar to that of the *Second Approximate Arrangement* heretofore exhibited, (10); the value of the *interior* half-planet limit ♅i and the standard value[2] of the asteroid limit (A) being both inserted; and besides the column of differences of Law from Fact in terms of the Earth's mean distance as 1, we have

would then approach more nearly to the standard value of limit (A). In this aspect of the matter, the difference of limit (A) from the mean in question would seem to be *on the right side.*

If, however, we take the mean between the two extremes of the known distances, that of Flora 2.20336, and that of Sylvia 3.49411 (as Prof. Kirkwood has done—*Proceed. of Royal Ast. Soc.*, vol. xxix. p. 99), we shall have the value 2.84873; which is almost exactly the same with the value of (A) here brought out.

[1] What ought to be the *mass* of the missing half-planet cannot be ascertained without the introduction of *theoretical* considerations; of which more hereafter.

[2] As exhibited in *Article* (12).

an additional column expressing in every case the same difference in terms of the quantity to be compared, which is *a′*, the planet's own mean distance from the Sun, or else *d′*, the distance from the Sun of the *limit* in question.

Thus, for example, in the instance of Saturn, Law—Fact = 0.094 of the Earth's mean distance; and *that*, in the next column, is seen to be only 0.010 of Saturn's own mean distance from the Sun.

COMPLETED ARRANGEMENT OF THE PLANETARY SYSTEM, EXHIBITING THE CORRESPONDENCE OF LAW WITH FACT.

TABLE (B).

Names and Symbols.				Law.	Fact.	Law—Fact.	
						Earth's dist. = 1.	a′ or d′ = 1.
♆		Neptune, $r^{\frac{3}{4}}$....		30.057264	30.057332	— 0.000 +	— 0.000 +
♅	$r^{\frac{1}{2}}$	*Uranus,*	r	19.55718	19.18336	+ 0.374 +	+ 0.019 +
(U)		Limit (U), ...		16.91431	....	....	
♅*i*		*Int. to* ♅,	r	(14.64275)	(missing)	....	
♄		Saturn, ...		9.44511	9.53885	— 0.094 —	— 0.010 —
			r				
♃		Jupiter, ...		5.23391	5.20280	+ 0.031 +	+ 0.006
			r				
(A)		Limit (A), ...		2.87831	(2.82293)	+ 0.055 +	+ 0.020 —
			r				
♂		Mars, $r^{\frac{3}{4}}$......		1.57096	1.52369	+ 0.047 +	+ 0.031
⊕	$r^{\frac{1}{2}}$	*Earth,*	r	0.99335	1.00000	— 0.007 —	— 0.007 —
(⊕♀)		Limit (⊕♀), ...		0.85101	....	....	....
♀	r	*Venus,*	r	0.72975	0.72333	+ 0.006 +	+ 0.009 +
Aph. ☿		Aph. of Mercury,		0.45758	0.46670	— 0.009 +	— 0.020 —
☿		MERCURY,	$r^{\frac{3}{4}}$	0.39166	0.38710	+ 0.005 —	+ 0.012 —
Per. ☿		*Per. of Mercury,*		0.28573	0.30750	— 0.022 —	— 0.071 —

The coincidences between Law and Fact, as compared with previous approximations, are now far more complete. The greatest actual difference is that in the instance of Uranus, which, after all, on the large scale of that planet's orbit is less than $\frac{1}{50}$th of the quantity to be measured.[1]

The distances of Mercury in aphelion and in perihelion as stated in the column of Fact are themselves computed from Mercury's mean distance and the eccentricity of his orbit, *at the present date.* With other values of the eccentricity, we would have had as follows:—

Eccentricity.	Aph. Dist.	L.—F	Per. Dist.	L.—F.
Maximum[2] = 0.2317185	0.47680	— 0.019 +	0.29740	— 0.012 —
Mean = 0.1766064	0.45546	+ 0.002 +	0.31873	— 0.033
Minimum[2] = 0.1214943	0.43413	+ 0.023 +	0.34007	— 0.054 +

[1] Why, after all, Uranus seems to have, as it were, *fallen in* from his appropriate position, may be considered in another connexion; not here, where only the relations themselves are permitted to have place, without the introduction of any physical hypothesis to explain them, as was indeed intimated in the first part of this Section. The same may be said of Mars.

[2] The maximum and minimum values of the eccentricity here inserted, are those given by John N. Stockwell, M.A., in his Memoir on the Secular Variations of the Elements of the Orbits of Eight Principal Planets, Introduction, p. xi.—*Smithsonian Contributions to Knowledge*, vol. xviii.

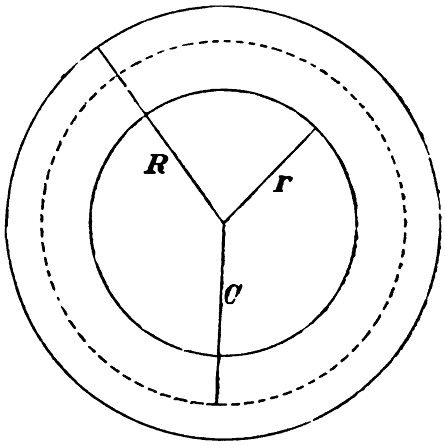
SATELLITE SYSTEMS.

System of Saturn.

(15) In the System of Saturn we find again three ratios; all of them fractional powers of one another, and one of these, like the special one in the Planetary System, *the square root* of another.

The rings, both bright and dusky, have also *their* places in the satellite series, with the condition always understood, that the distance of any ring from Saturn's centre is to be measured from that ring's *own centre of gyration*.

Fig. 1



(16) Now the *centre of gyration* of an *indefinitely thin* ring, and one which has, in effect, a uniform density and thinness, this centre, has *itself* special relations which it will be well to notice.

For let R be the radius of the outer edge of the ring, C the distance of the centre of gyration from Saturn's centre (or from the common centre of all the circles in question), and r the radius of the inner edge of the ring.

Then, we have

$$C = \sqrt{\frac{R^4 - r^4}{2R^2 - 2r^2}}.$$ [1]

or,

$$C = \sqrt{\tfrac{1}{2}\cdot\frac{R^4 - r^4}{R^2 - r^2}}.$$

That is

$$C = \sqrt{\tfrac{1}{2}\cdot\frac{(R^2 + r^2)\,(R^2 - r^2)}{R^2 - r^2}};$$

or

$$C = \sqrt{\tfrac{1}{2}(R^2 + r^2)} \quad . \; . \; . \; . \quad (A).$$

But now, if the ring be supposed to be *so* divided by the circumference of a circle concentric with the edges of the ring, that the two portions thus obtained shall be *equal in area*, and the radius of this bisecting circumference be x; then the expressions for the two portions of the ring will be equivalent to one another, and so we shall have

$$\pi(R^2 - x^2) = \pi(x^2 - r^2);\ \text{whence}$$
$$R^2 - x^2 = x^2 - r^2;\ \text{and}$$
$$R^2 + r^2 = 2x^2;\quad \text{whence}$$
$$x^2 = \tfrac{1}{2}(R^2 + r^2);\ \text{and}$$
$$x = \sqrt{\tfrac{1}{2}(R^2 + r^2)} \quad . \; . \; . \; . \; . \quad (B).$$

[1] Dr. Olinthus Gregory's Mechanics, 4th edition, Art. 312, Ex. III.

The value of x in equation (B) is the same with that of C in equation (A). Hence

$$C = x;$$

or *the centre of gyration is in the circumference of a circle concentric with the edges of the ring, and bisecting its area.*

And a cylindrical surface having this bisecting circle for one of its edges, and cutting perpendicularly through a ring formed like that of Saturn, would (density uniform) also bisect the volume of the ring, and also would bisect the material of the ring; and the value of C, the centre of gyration of this ring of sensible thickness, would not be affected by these new circumstances; the indefinitely thin ring being the plane of rotation on which the other might be projected.[1]

(17) The equation for the centre of gyration of any two equal masses will take the same form as that of *Eq.* (B), with the condition, however, that R and r shall respectively denote the radii of gyration of those masses. Indicating these radii then by R' and r', and the masses (equivalent or not) by M and m; and then (since velocities are as radii of *simultaneous* rotation) the general formula will be thus expressed:—

$$C = \sqrt{\frac{MR'^2 + mr'^2}{M + m}} \; \ldots\ldots \; \text{(C)};$$

which, when $M = m$, is reduced to

$$C = \sqrt{\tfrac{1}{2}\,(R'^2 + r'^2)} \; \ldots\ldots \; \text{(C)}',$$

so that when the equivalent masses are both rings, the one wholly clasping the other, like the two halves of the ring in question, the position of the centre of gyration may be obtained by a similar process, whether the $\frac{1}{2}$ sum of the squares under the radicle be that of those quantities representing the radii of outer and inner perimiters of the whole ring, as in *Eq.* (A); or the radii of gyration of the respective halves, as in *Eq.* (C)′.

---

[1] This property of the centre of gyration of a ring like those of Saturn, as well as of the indefinitely thin ring, has about it a species of mathematical elegance. I know not whether the enunciation of it is new; but the correspondence of the position assigned by it with that of the division between the bright ring systems of Saturn, is a curious, if not an interesting one. [See *Article* (19).]

SYSTEM OF SATURN.

TABLE (C).

(18) *Definite Arrangement of the System.*

NAMES, etc.			LAW.	FACT.	DIFF. L.—F
Japetus,	r'		64.3590	64.3590	0.00
....	r'	r'^4			
....	r'				
....	r'				
Hyperion,	r'	r'	27.4069	26.7834	+ 0.62 +
Titan,			22.1397	22.1450	— 0.01 —
....	r	r^2			
....	r				
....	r'	r'			
Rhea,	r''	$r\frac{1}{2}$	9.5972	9.5528	+ 0.04 +
....	r''	$r\frac{1}{2}$			
Dione,	r'	r'	6.8453	6.8398	+ 0.01 —
Tethys,	r	r	5.3365	5.3396	— 0.00 +
Enceladus,	r'	r'	4.3109	4.3135	— 0.00 +
Mimas,	r	r	3.3607	3.3607	0.00
	r	r			
....	r'	r'			
Outer B. Ring,	r'	r'	2.1165	2.1246	— 0.01 —
Inner B. Ring,			1.7097	1.7323	— 0.02 +
Dusky Ring,	r'	r'	1.3811	1.3402 1.3588	+ 0.04 + + 0.02 +

In the instance of the Dusky Ring two values appear in the column of Fact; the first of these indicating the position of the centre of gyration, if the Dusky Ring have *an interval* between it and the inner Bright Ring (proportional, perhaps, on a smaller scale, to that which exists between the two systems of Bright Rings). The second value is that which obtains, if we suppose the Dusky Ring to extend quite up to the Bright Ring. The difference between the results is but a small fraction of the quantity to be compared.

[In view of the very considerable number of limits in the upper region of the system at which no satellite is found, and the ratios themselves being so small, it might almost seem that the approximate coincidence between Law and Fact was a forced one, brought about by a special arrangement and combination of terms. But not merely the number of terms (or ratios, or their equivalent) is indispensable, but the right order of their grouping must also be measurably maintained, to bring about the coincidences in their appropriate places. Then, afterward, from Dione downward, every limit has its corresponding satellite or ring, with the bare exception of that between the satellites and the rings. Then the discrepancy between Law and Fact is, in most cases, all but insensible. The most conspicuous deviation is that in the instance of the more recently discovered satellite Hyperion, the distance of which is not yet well determined. Another fact seems also not without its significance; viz., that the two ratios in the region of the rings have the same value, r'.][1]

The somewhat abnormal deviation from Law in the instance of *Hyperion*, presents a case like those of *Uranus* (especially) and, also, *Mars*, in the planetary system;[2] the resemblance being all the more accurate because the difference from Law is, in all these instances, *negative*. These, and other peculiarities, will be reviewed in the aspect of theory, in Section III.

Other Relations.

(19) The centre of gyration of the whole system of Bright Rings is at the distance from Saturn's centre $= 1.9090$; being just within the outer edge of the inner Bright Ring (or Rings) which is at the distance 1.9276.

In the subordinate system of the two outer Bright Rings the ratio of their distances (2.1825 – and 2.0522 –) $= 1.06438$; while $r^{\frac{1}{4}} = 1.06423$.

Manifestly, then, the arrangement of the *Outer System* of Bright Rings is

$$\left.\begin{matrix}\text{Exterior Ring}\\ \text{Interior Ring}\end{matrix}\right\} r^{\frac{1}{4}}, \text{ agreeing well with } \overset{\textit{Fact.}}{\left\{\begin{matrix}2.1825 -\\ 2.0522 -\end{matrix}\right\}}$$ [3]

System of Jupiter.

Table (D).

(20). *Definite Arrangement of the System.*

Satellites.	Law.	Ratio.	Fact.	L.—F.
IV.	26.99835	$r = (1\ 6007)^{\frac{6}{5}}$ (IV.–III.)	26.99835	0.000
III.	15.35202		15.35024	+ 0.002 –
II.	9.62147	$r' = 1.5956$ (III.–II.)	9.62347	– 0.002
I.	6.04934	$r' = 1.5905$ (II.–I.)	6.04853	+ 0.001 –

Here $r = r'^{\frac{6}{5}}$, or $r' = r^{\frac{5}{6}}$; and the value of r' regularly *diminishes* by 0.0051.

[1] The accepted values in the column of Fact agree very closely with the very careful deductions of Capt. Jacob, from his own observations (*Memoirs of the Royal Astronomical Society*, vol. xxviii. p. 108). These are referred to Titan's distance as the standard; and when measured by Saturn's eq. radius give for

Rhea	9.5562	instead	of	9.5528.
Dione	6.8445	"	"	6.8398.
Tethys	5.3470	"	"	5.3396.
Enceladus	4.3207	"	"	4.3125.

[2] See *Note* 1 to (14).

[3] Of these relations, and what else is connected with them, more hereafter in Section III.

SYSTEM OF URANUS.

TABLE (E).

(21) *Approximate Arrangement.*

Satellites.	Mean Distance from Planet.	Ratios.
Oberon	22.56	1.3333
Titania	16.92	$(1.3913)^{\frac{3}{2}} = 1.6411$
Umbriel	10.32	1.3932
Ariel	7.40	

Here $r = r'^{\frac{3}{2}}$, or $r' = r^{\frac{2}{3}}$; and the value of r' *increases*; as r did (but regularly) in the planetary system.

Summing up of Relations of Mean Distances from their Respective Centres.

(22) In the *Planetary System* the value of the leading ratio r is at first 1.7770, and the regularly progressive *increase* of its value afterwards, from term to term $= 0.0138$. Also $r' = r^{\frac{3}{4}}$; and $r'' = r^{\frac{1}{2}}$.

In the *System of Saturn* $r = 1.28273$, $r' = r^{\frac{6}{7}}$, and $r'' = r^{\frac{1}{2}}$; and all the ratios are *constant*. Moreover, for the two outermost rings, $r''' = r^{\frac{1}{4}} = (r'')^{\frac{1}{2}}$.

In the *System of Jupiter* we have $r' = r^{\frac{3}{4}}$; r', at first, $= 1.6007$; and the regularly progressive *decrease* of its value $= 0.0051$.

In the *System of Uranus* $r' = r^{\frac{2}{3}}$; and the value of r' shows an *increase* from term to term.

Additional Feature of Resemblance of Two Half-Planets.

(23) The inclination of the equator of *Venus* to the plane of that planet's orbit, does not seem to have been accurately determined, but it is usually stated to be nearly 72°; the rotation of the planet (as is usually the case) being direct.

In the *Monthly Notices of the Royal Astronomical Society*, vol. xxiii. p. 166 (Jan. 1873), *W. Buffham, Esq.*, as a merely approximate result as yet, makes the inclination of the equator of Uranus 80°.[1] "Movement direct."

The orbits of the satellites are inclined to the ecliptic at an angle of about 79°; and *their* motion is *retrograde.*

These two half-planets, then, though near to the two extremes of the system, are again alike; viz., in the great inclinations of their equators, as well as in the direction of their rotations.

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[1] Inclination, viz., to the plane of the ecliptic. The inclination to the plane of the planet's own orbit is about 79⅓°.

SECTION III.

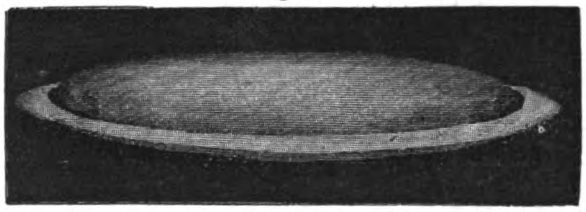
APPLICATION OF THEORETICAL CONSIDERATIONS AND THE DEVELOPMENT OF OTHER RELATIONS.

(24) The further discussion of the relations exhibited in Section II. will be aided, and circumlocution, at the same time, avoided, by the introduction of considerations having reference to the *Nebular Hypothesis of Laplace;* and this especially in the exposition of other relations, the investigation of which was prompted by suggestions furnished by the application of this very hypothesis somewhat extended and modified, in a manner now to be specified.

In the exposition of his hypothesis, its illustrious author supposes the atmosphere of the *rotating* Sun to have extended, in ancient times, to the limit (or, when at the furthest, very near to the limit) at which the centrifugal force of rotation must have balanced the force of attraction.

That afterwards—the atmosphere shrinking from loss of heat—the rotation (for reasons which he specifies) would be accelerated as the atmospheric molecules drew nearer to the centre of the Sun,[1] and, that the limit in the plane of the Sun's equator, at which the two forces—centripetal and centrifugal—would balance one another, would, therefore, be found further and further in.[2]

Fig. 2.



That thus successively, at new limits *in the plane of the Sun's equator*, further and further inward, the centrifugal and centripetal forces would indeed balance one another; insomuch that the *thin and narrow zones* thus *in equilibrio* in the plane of the equator (they having no tendency either to fall in or to be thrown off), would themselves be "abandoned" by the atmosphere in its farther shrinkage.[3]

(25) The description then goes on to state that the same equilibrium of forces *not* existing with respect to the atmospheric molecules situated on the *parallels* to

---

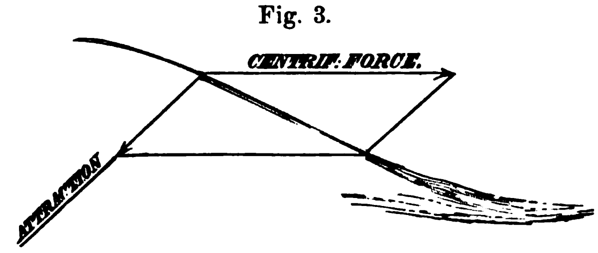
[1] The loss of heat will not affect the moment of rotation—the *turning power*—and every molecule (because of the shrinkage) having a shorter circuit, will accomplish it in *less time*. Then also, as shown hereafter, there will be some acceleration of the *actual velocity*. The original phraseology, as it were, anticipates this also, and provides for both. "La rotation doit être plus prompte, quand ces molecules se rapprochent du centre du soleil."—*Exposition du Système du Monde*. Note VII

[2] The centrifugal force, in accordance with its law, increasing at a more rapid rate than the attractive force; the centrifugal force (with conservation of areas) varying inversely as the cube of the distance, instead of inversely as the square of the distance, so that, at a distance *a little within* the atmospheric limit, and at which the attractive force was still somewhat *in excess*, it would soon happen that a small increase of both forces (from the shrinkage of the material) would result in increasing the centrifugal force so much more rapidly as to exhaust the difference of the two forces, and leave the nebulous material ready to be "abandoned."

[3] Very different this, from the supposition of many misinformed persons, that the rings here spoken of were *thrown off* by an excess of centrifugal force.

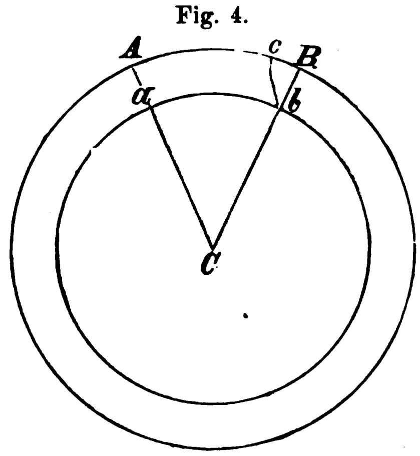
the solar equator, these molecules would, by their attraction, be brought closer to the atmosphere, in the progress of its condensation, and would not cease to belong to it until, in consequence of this motion, they were brought nearer to the plane of the equator.[1]

Fig. 3.



(26) The description proceeds, saying of these "zones of vapor" (or rather nebulous zones) successively abandoned, that these zones, must, in all probability, form by their condensation and the mutual attraction of their molecules, diverse concentric nebulous rings circulating around the Sun. The mutual friction of the molecules of every ring must accelerate some and retard others, until all had acquired the same angular motion. And (when all this went round together) the actual velocity of molecules further from the centre would be greater than that of those nearer; the parts near the outside of the ring going *uniformly* round in a large circuit, in *the same time* in which those nearer, also moving *uniformly*, described a smaller circuit. Thus, with time the same, the angle ACB being the same for both, the part, such as AB, is greater than the similar part ab of the smaller circuit;[2] and the *part* of AB described in a unit (say a second) of time, greater than the similar part of ab; *i.e.* the *actual velocity* in AB is *greater.*

Fig. 4.



(27) Besides all this, in the progress *inward* of the particles forming the nebulous rings; the *actual velocity of rotation* of those particles would be *increased* conformably to the principle of the conservation of areas; which requires that an area such as ACB, in the figure, should *continue* to be passed over, by the rotation of CB, in the *same* time; so that if AC and BC be shortened, the figure must be *broader* to preserve its size, or the distance BA, traversed in *the same time* must be *greater* than before; *i. e*, the particle must move *faster* along BA; while the particles attracted toward the others *outward*, and then forming the inner part of the ring, would, in obedience to the same principle, have *their* actual velocity of rotation *diminished.*

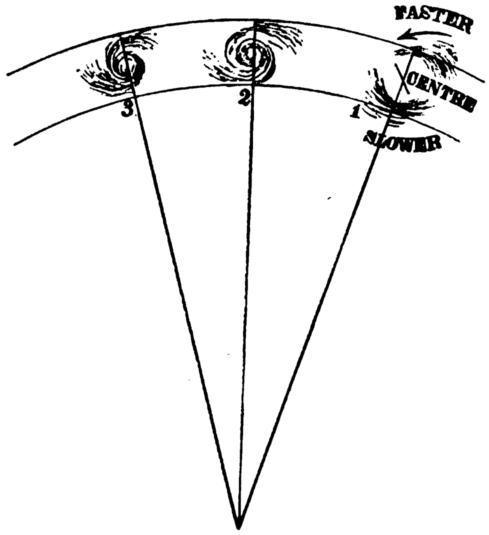
(28) Then if all the molecules of the nebulous ring continued to condense without being disunited, they would at length form a liquid or a solid ring.[3] But the regularity requisite in such a case, in every part of the ring and also in its cooling, must make this a very rare phenomenon. Accordingly the solar system affords but a single example of this kind—that of the rings of Saturn.

[1] The diagrams are our own. M. Laplace employs none in his Exposition du Système du Monde.
[2] The difference being $= Bc$.
[3] Or a ring of small solids closely arranged, as seems to be actually true of the rings of Saturn.

(29) But almost always, the nebulous ring must have broken into several masses, which, moving with velocities but slightly different, would continue to circulate at the same distance from the sun.

These masses would take a spheroidal form with a motion of rotation in the direction of their motion of revolution (from west to east), because of the inferior molecules (26), having less actual velocity than the superior; and thus would soon be formed so many nebulous planets. But if one of these were sufficiently powerful to bring together successively, by its attraction, all the others about its own centre, the nebulous ring would then be transformed into a single nebulous spheroidal mass revolving around the sun, and having a rotation in the direction of its revolution. This last has been the most common case; though the solar system, nevertheless, furnishes an example of the first case, in the small planets which revolve between Mars and Jupiter, at least if we do not suppose with Olbers that they primitively formed a single planet, which a powerful explosion divided into several parts animated with different velocities.

Fig. 5.



(30) Now if we follow the changes which an ulterior cooling would produce in the nebulous planets of which we have come to conceive the formation, we shall see form, at the centre of each, a nucleus incessantly increasing by the condensation of its surrounding atmosphere.

(31) In this state the planet would perfectly resemble the sun in the nebulous state in which we considered it. The process of cooling must then produce, at different limits in its atmosphere, phenomena similar to those which we have described; that is to say, rings and satellites circulating around its centre in the direction of the planet's own rotation, and turning at the same time (the satellites that is) upon themselves. The regular distribution of the mass of the rings of Saturn about its centre, and in the plane of its equator, results naturally from this hypothesis, and without it becomes inexplicable. "The rings" (exclaims the framer of the hypothesis) "appear to me to be an ever-present proof of the primitive extension of the atmosphere of Saturn, and of its successive retreats."[1]

(32) He then proceeds to say that the singular phenomena of the small eccentricity of the orbits of the planets and the satellites, of the small inclination of those orbits to the solar equator, of the identity of direction of rotation and revolution of all[2]

[1] "Me paraissent être des preuves toujours subsistantes de l'extension primitive de l'atmosphère de Saturn, et de ses retraites successives."

[2] Difference of density, etc. might cause the rotation of a satellite in a rare case to be in a contrary direction, as is true of the orbital motion of the satellites of Uranus.

these bodies with that of the rotation of the sun, flow from the hypothesis which he proposes, and give to it great probability.[1]

(33) If the solar system had been formed with perfect regularity, the orbits of the bodies which compose it would have been circles, the planes of which, as well as those of their several equators and rings, would have coincided with the plane of the solar equator. But we may conceive that the innumerable varieties which must exist in the temperature and density of the different parts of those great masses, have produced the eccentricities of their orbits, and the deviation of their motions from the plane of that equator.

(34) The author then goes on to show that, on this hypothesis, the comets are strangers to the system, formed by the condensation of nebulous matter elsewhere, but drawn in when they come into the region in which the attraction of the sun is predominant; and he then proceeds further to show that this will account for all the peculiarities of their motion, as well as the variety in the inclinations of their orbits.

(35) M. Laplace then adds that, if in the zones abandoned by the atmosphere of the sun there were found molecules too volatile to unite to one another, or to the planets, they ought, while continuing to circulate around the sun, to present all the appearances of the Zodiacal Light, without opposing sensible resistance to the several bodies of the planetary system, either because of their extreme rarity, or because their motion is the same with that of the planets themselves.

(36) In all that has now been stated, which, for the most part, is a translation, or else a paraphrase of M. Laplace's *Note VII.* to his *Exposition du Système du Monde*, in all this, there has been no allusion to the operation of another cause, which may well have produced changes in the nebulous material, antecedent to those which have been already contemplated. The solar atmosphere, when at its largest extent, must also have had a very oblate form, and the portions near to the pole of the rotating sun, because of the superior density, and close proximity of the sun's body, have been subjected to an attractive force greatly superior to that prevalent (or barely in equilibrio) in the equatorial regions.

(37) Now a greater attractive force acting on nebulous matter increases the local density where the force is thus urgent; as is manifest from what we observe in the nuclei of comets. But a greater density of the same sort of material is accompanied by *a more profuse radiation of heat.* All this could not fail to produce changes in the actual, as well as angular, velocity of the portions thus affected, which would not conform to the changes of both, then going on, in the regions nearer to, or at the equator.[2] A *rending* of the material of the atmosphere must thus result, perpetuating itself all round the sun, so long as the portions most affected were not detached to the extent of "abandonment."

There might still be a tendency in the portions thus separated by the rent from those parts still closely attached, to preserve, at least rudely, an approximation, even in their exterior surface, to the spheroidal form; the situation, at any given distance from the axis—when once that situation has been attained—presenting the same ratio there of centripetal and centrifugal forces; since, in so far as density

[1] Verisimilitude rather—"vraisemblance."

[2] To say nothing of the *molecular* changes which might be superinduced by the condensation itself.

is concerned, the centrifugal force at the extremity of the radius of rotation, would be as the density, and the attractive force, still acting at the same angle with the plane of the parallel, be also as the density, so that the element of density being, in effect, all but excluded from the comparison, there would remain very nearly the same ratio of the forces as before; so that the not yet "abandoned" portion of the atmosphere would scarcely have its exterior spheroidal form affected.[1]

And, although the case is not *just the same*, divisions into something like *spheroidal shells* resembling those here supposed may be[2] traced in the representations of the heads of comets, among others that of 1680, as represented in Plate VI. of the third volume of *Delambre's Astronomie Théorique et Pratique;* the same being copied from the *Histoire Céleste* of *Lemonnier.* The appearance in question is yet more conspicuous in the representations of the head of the great comet of 1858, given by Prof. G. P. Bond. in Vol. III. of the *Annals of the Observatory of Harvard College.* A very faithful copy of one of these is here given.

Fig. 6.

(38) Now, the partially condensed *shell* thus formed (if indeed admissible) must itself have exerted a *conservative power* in preventing the too frequent occurrence of cases like that of the asteroids; viz., by an earlier holding together of the greater number of the "abandoned" equatorial portions of the atmosphere in the process tending to form rings or planets.[3]

Nay, it might even be questioned whether the more dense portions of the atmosphere, earlier separated, may not in their progress toward the equatorial plane, described in (25), have arrived at the state of equilibrium of the forces, before the equatorial portions were ready for the same; and so, *the formation of a planet have gone on thus far, from a shell instead of a ring.*

Just one change more, to be followed by its consequences, might then have taken place. The more dense portions, being the *first* about to be "abandoned," might be found to be further *outward* than the rarer equatorial portions; and attaching the latter to themselves by the attraction due to a greater density.

(39) Now, the special arrangements of the two half-planets, Earth and Venus, are *as though* what has here been discussed and explained, were entirely applicable to them.

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[1] Though the ellipticity of the same might be appreciably changed.

[2] Which may indeed, in part, be consequent on the changes adverted to in *Note* 2, on p. 20.

[3] The oblate form of the spheroid here alluded to; the more profuse radiation of heat due to a greater condensation of the nebulous material in the polar region; and the division of the envelope into shells were all insisted upon by the author of this paper in a communication made by him to the American Association for the Advancement of Science, at their meeting in Montreal, in 1857. The idea of a more profuse radiation of heat from the polar regions seems, since that date, to have independently occurred to others; and a profound and thorough investigation of the form of the oblate solar spheroid and its variations, as also of the density of the solar atmosphere, at the various planetary distances, the relative breadth of the rings, etc., though without reference in that connexion to a more profuse polar radiation, is given by David Trowbridge, A.M., in vol. xxxviii. (Second Series) of the *American Journal of Science and the Arts,* Nov. 1864.

Specialities of the Half-Planets Earth and Venus.

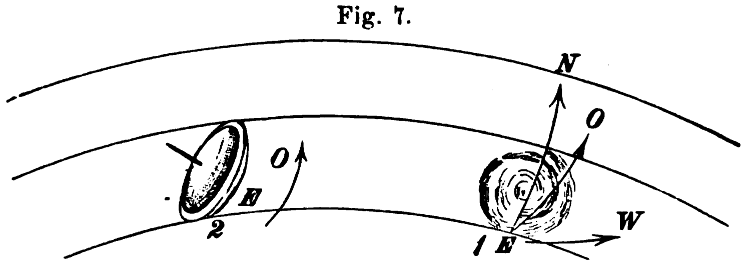
1. In accordance with the immediately preceding conclusion, the *exterior* half-planet, the Earth, not merely shows a density greater than that of its interior half-planet Venus, but also, as seen in Table (A), in (3), a density altogether remarkable in view of the Earth's place in the planetary system.

2. The inclination of the equator of Venus to the plane of that planet's orbit (from 73° to 75°, most probably) presents a marked contrast to what we find in the cases of Mercury, the Earth, and Mars, in all which the inclination of the equator approaches to a mean value that is nearly the same with the obliquity of our equator to the ecliptic; and this, while a like contrast does not exist in the respect of the time of rotation (the sidereal day) of Venus; for that is nearly the same with each of the respective sidereal days of these same other three planets, in this region of the system. But the inclination of the equator of Venus is, up to the present time, without a parallel in all the system, except in the instance of another *half*-planet, viz. Uranus.[1]

And here the state of things is, withal, *as though* the enormous deviation of the plane of the equator from the plane of the planet's own orbit (and which implies also a very large deviation from the plane of the sun's equator) were itself *due to the attraction* towards the more dense outer portion, already commented on, which went to the formation of the Earth; an attraction acting in a direction nearly *perpendicular* to the half-planet's first-forming equator and its parallels.

Thus the material, at its first rolling up from the form of a ring or shell, would be inclined to rotate in the plane of *EW*, but being drawn outward by the attraction of the more dense material in the direction *EN*, the resultant rotation would be in a direction such as *EO*, as represented in the figure at 1, and transferred to the position marked 2.

Fig. 7.



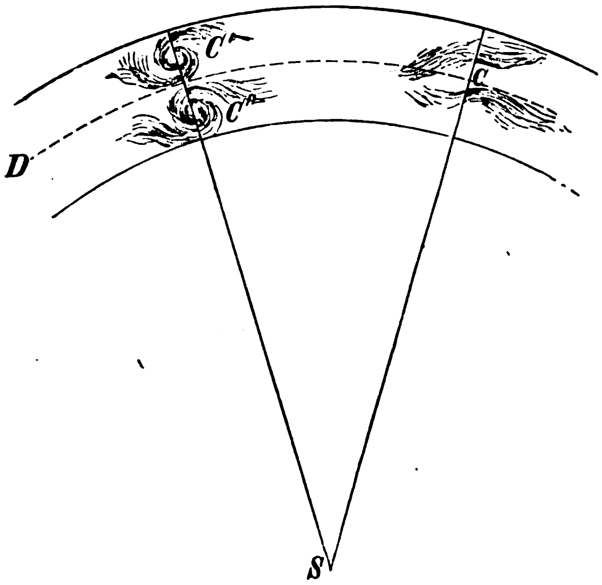
All this might begin antecedently to the process of rending which introduced the formation of half-planets, or perhaps go on during that very process; in which

[1] During the *revolution* of a whole ring or shell around the sun, every part of the outside would be presented *once* in its turn to the entire circuit of the heavens; and so in effect would *rotate once* around a point within that ring or shell. This would determine the angular velocity of rotation at the first gathering up to form a planet. The existence of more dense material outside would seem *not* to have superinduced a *retrograde* rotation in this case; but to have interfered to the preventing of an *accelerated* rotation, and thus the *more dense* material be kept *outside*, until, in the contest of forces, the rending into two half-planet masses took place. The existing state of things, in its various aspects, seems to look toward this; but the problem is too complicated a one to justify an assertion that such was the succession of events.

same process of rending, the attraction of material outward, *i.e.*, toward the more dense Earth-forming mass, may itself have been efficient.[1]

3. The division of material into two half-planet portions, would very probably take place, at what, with reference to the revolution around the sun, was the *centre* (or rather the central line) *of gyration* of the whole mass (at the distance SC in the figure); leaving the material on the one side and the other of that limit, to be gathered into the half-planet masses, each around its own special centre of gyration (at C' and C''); which special centre would be that due to the *half-planet itself*, when formed.

Fig. 8.



Making use, then, of the half-planets themselves (gathered at C' and C''),[2] and finding *their* centre of gyration, we shall approximate to the former position of (C) the centre of gyration of the whole mass. But *that* would be the position of the *whole* planet, if the material had all gone to form it, *i.e.*, the *limit* (⊕♀) in Table (B), so that the centre of gyration of the two half-planets should be found very near to the limit (⊕♀) in Table (B), in (14).

Now—with the masses of the Earth and of Venus as given in Table (A), in (3), and their distances as given in the column of Law in Table (B) in (14)—from *Eq.* C in (17), we have for the distance from the sun of the centre of gyration of the Earth and Venus,

with sun's horizontal parallax $= 8''.848$, $C = 0.88665$
" " " " $= 8.78$, $C = 0.88579$.

And the position due to the *whole* planetary limit (⊕♀) in Table (B), in accordance with *Law* 1*st* (10), is

$$(\oplus ♀) = 0.85101.$$

4. But the separation of the material into two half-planet portions would, withal, take place at the limit where the attractive forces of the forming half-planets were in equilibrio; on one side of which limit the material would be gathered (by the excess of attractive force on *that* side) in the formation of a half-planet toward *that* side; and on the other side of (the neutral) limit, in the formation of another

[1] A writer in the *Westminster Review*, vol. lxx. (July, 1858), has introduced the idea of a greatly inclined rotation in a *thick* ring, or even a retrograde rotation; but he has applied it in a region of the system in which the conditions which he introduces are misplaced. A different explanation is applicable in the instance of Uranus, as will be shown hereafter.

[2] Which will scarcely differ, in either case, from the very centre of the planet itself.

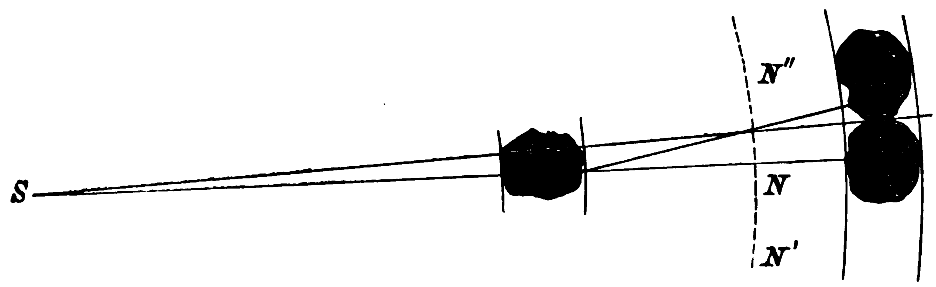
half-planet on that *other* side [as they are represented in Fig. 8], gathering around C', C'', the one on the one side, and the other on the other side of CD, the dividing limit of neutrality, where the forces being equivalent and opposed would be in equilibrio. It would seem then to be desirable to ascertain whether the limit *thus* defined will agree with either, or nearly with both, of the other two determinations already made.

Fig. 9.

Now when two planets (P and P') are in conjunction, as seen from the sun (at S), the position of the point (N), at which their attractions would be equivalent and opposite, and so neutralize one another, may be found, as is well known, by so dividing the distance (PP') between those planets, that

$$\frac{NP}{NP'} = \frac{\sqrt{\text{of mass of } P'}}{\sqrt{\text{of mass of } P}}.$$[1]

Fig. 10.



But, in the act of the rending described in the *Note* on p. 22, portions such as Q and Q' would act on one another directly (in the line QQ') very much as would two small planets; and so the neutral point (N) be determined as before, viz.:—

$$\frac{QN}{Q'N} = \frac{\sqrt{\text{of mass of } Q'}}{\sqrt{\text{of mass of } Q}},$$

And the local oblique action of neighboring portions would conform to very nearly the same ratio; so that the whole action within distances at which it would be appreciable would have its neutral limit ($N'NN''$) dividing the distance between the points of reference of rupturing annular masses in a manner approximating to that which obtains in the case of two planets. And what is here stated of them, might also be asserted of the sections of shells, parallel to the equatorial rings, with approximately the same result as to the dividing limit.

Making use then, as heretofore, of the half-planets themselves, as accumulated around what were their respective points of reference, while yet their masses were

[1] The point N is one of the limits of Prof. Kirkwood's spheres of attraction, made use of in his Analogy.

in the former state; we shall, by the application of the equation here adopted, in effect obtain QN or $Q'N$, and hence also SN, the distance of the neutral point N from the sun's centre. With the same data from Tables (A) and (B) in (3) and in (14), as before, we shall then have

with the sun's horizontal parallax 8″.8448,	$SN =$	0.85383,
and with " " " 8.78 ,	SN	0.85459.
While, (14), limit (⊕♀) due to a whole planet distance in Table (B), is		0.85101,
exhibiting all but a perfect coincidence; while, as before, the distance of the centre of gyration from the sun's centre . .	$SC =$	0.88665, or 0.88579,

(40) Summing up then the specialities of the two half-planets, Earth and Venus, which are consistent with the theoretical considerations now exhibited, we have

1. In accordance with the conclusion in (39), the greater density of the *exterior* half-planet, the Earth.

2. The *tilting up* (if the expression be allowable) of the equator of Venus and its parallels—as if by the attraction outward, due to that same greater density—in the antecedent arrangement of the half-planet masses.

3 and 4. The decided approximation to agreement in position of—

(*a*) The whole planet limit (⊕♀) in Table (B).

(*b*) The neutral point, or point of equal attraction between the two half-planet masses, and

(*c*) The distance from the sun's centre of the centre of gyration of the same two half-planet masses, thus—

$$(\oplus ♀) = 0.851 +$$
$$\text{Neutral position is at } 0.854 \pm$$
$$\text{Centre of gyration is at } 0.886 \pm.$$

Determination of the Mass due to a Half-Planet ♅i (now missing), interior to Uranus.

(41) The distance due to such a half-planet has already been determined in accordance with *Law* 3*d*, (10), and the same is recorded in Table (B), in (14).

The *mass* of this half-planet may be determined by means of the equation for the centre of gyration of it and Uranus; the case being similar to that of the Earth and Venus,[1] and the whole planet limit here being *limit* (U), in Table (B).

Now let a' represent the mean distance of Uranus from the sun, and m' the mass of that planet; while a and m, respectively, represent like quantities in the instance of $♅_i$. Then, as limit (U) represents the position due to the centre of gyration, *Eq.* (C) of (17), will read

[1] But here the agreement of the position of the centre of gyration with the whole planet limit, will have this favoring condition; that under the less stringent circumstances, in this region of the planetary system, it is not probable that any considerable portion of the more dense material was carried to the *outside*, in the half-planet formation (or the tendency to it), as, (39), seemed to have been true in the instance of the Earth.

$$(U) = \sqrt{\frac{m'.a'^2 + m.a^2}{m + m'}}; \text{ or}$$

$$(U)^2 = \frac{m'.a'^2 + m.a^2}{m + m'}; \text{ whence}$$

$$m(U)^2 + m'(U)^2 = m'.a'^2 + m.a^2; \text{ and}$$

$$m \left\{ (U)^2 - a^2 \right\} = m' \left\{ a'^2 - (U)^2 \right\}; \text{ and}$$

$$m = \frac{a'^2 - (U)^2}{(U)^2 - a^2} \times m'; \text{ or}$$

$$m = \frac{\overline{a' + (U)} \times \overline{(a' - (U)}}{\overline{(U) + a} \times \overline{(U) - a}} \times m';$$

which, as a', (U), and a are all determined, will give us m in terms of m'.

Substituting, then, the values of a', (U), and a, as found in the column of *Law* in Table (B), in (14), we have

$$m = (1.38865)\, m',$$

i. e., *the mass of* ♁i = (1.38865) of the *mass of Uranus;* or, substituting the value of the latter, as found in Table (A), in (3), we shall have

$$\textit{Mass of } ♁_i = \tfrac{1}{15843} = 0.00006312 - \text{ of the mass of the sun.}$$

The most probable Answer to the Question—What has become of the Missing Mass?

(42) The most ready reply to this question would seem to be—that the missing mass had, (29), been formed into a group of *asteroids*. But then, as this region of the planetary system is one in which large masses abound, it would also seem that the mass of a group of asteroids here, might reasonably be supposed to be very considerable, even if the computation already made, (41), had not indicated this very mass to be almost $1\frac{4}{10}$ that of *Uranus*.

And if these considerations are conceded to have weight, the existence of the seemingly missing mass, in the form of a group of asteroids, becomes at once inadmissible; since, if such a group were there, its existence would speedily be evidenced by the perturbations of both Uranus and Saturn, which such a group would produce.

(43) Rejecting, then, the hypothesis of the existence of a group of asteroids in this region, the next hypothesis which it may be found to be appropriate to consider will be, whether, in the accumulation of the great mass which was to constitute *Saturn*, the material which would have formed the *interior* half-planet ♁i was not *itself drawn over and inward by the o'ermastering attraction of the Saturn-forming mass*, which thus *attached to itself* the interior half-planet mass rent away from Uranus.

In favor of this hypothesis we shall find *ten special consistencies*, which in their turn will *introduce others*, having *more extended relations*.

1.

The *mass* of the forming Saturn would be *adequate* to the exercise in its own place of the o'ermastering attraction here supposed.

For if from the mass of Saturn, as found in Table (A) in (3); viz.:—

$$\frac{1}{3501.6} = 0.00028558 +,$$

we subtract the mass of ⛢*i* $= 0.00006312 +,$

as computed in (41), there will remain $0.00022246 +,$

for the mass of the forming Saturn; *before* the mass due to the interior half-planet ⛢*i*, had been drawn over and inward to unite with the other portion of the entire mass which has gone to constitute the complete Saturn system as we now have it.

Now as the symbol for Saturn is ♄, we may represent this first *formative portion* of that planet's mass [which we just now found to be = 0.00022246 +] by the symbol ♄̂. And then computing the position of the point of equal attraction, or neutral point [as, heretofore, (39), in the case of Earth and Venus], we shall find ♄̂'s attraction to extend in the direction of Uranus, to the distance from the sun's centre = to 16.40924, which is far beyond the distance due to the (missing) interior half-planet ⛢*i* (viz., 14.64275) as found in Table (B), in (14). The attractive force of the pre-existing Saturn-mass was, then, adequate *in measure* to the effect here supposed.

2.

But this same limit, 16.40924, to which the attractive force of ♄̂ extended, in the direction of Uranus, this, also, is not so *very far short of* the limit (U),[1] *i.e.*, 16.91431, at which the whole planet mass would be likely to be rent to form the two half-planets, Uranus and ⛢*i*; it being, *in that respect*, a limit analogous to that found to be a dividing limit in the case of Earth and Venus in which *both* the half-planets *still exist*

3.

The very great inclination of the satellite system of Uranus to the plane of the planet's orbit was, long ago, determined by Sir William Herschel; the inclination of the orbits of the satellites to the plane of the ecliptic being nearly 79°; and the inclination to the plane of the orbit of Uranus must therefore be nearly 79°1',[2] while their ascending nodes on the ecliptic are nearly in longitude 166½°; *motion retrograde.*

And, again, the recent observations, (23), of W. Buffham, Esq., detailed in the *Monthly Notices of the Royal Astronomical Society*, vol. xxxiii., No. 3 (Jan. 1873), lead to results at present stated by him to be "the merest approximations;" but which yet give

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[1] In Table (B), in (14).

[2] Or 100°59'; the motion being *retrograde*.

Long. of the asc. node of the equator	110°
Inclination of the equator	80°
Time of rotation	$12^h \pm$;

motion direct.

From these several data, it would seem probable that the equator is inclined about $79\frac{1}{3}$° to the plane of the planet's orbit, and some 60° to the orbits of the satellites.

So that the drawing over of material (*inward* now, and not outward) due to the proximity of *the great mass of* ♄, would seem to have produced *in the direction of the plane of the equator of Uranus, an alteration like that which,* as heretofore shown, (39), *seems to have taken place in the instance of another half-planet, Venus;* the *tilting-up* (if the expression may again be tolerated) being quite as great in this instance as in the other; and here the orbits of the satellites are also enormously displaced.

4.

In the instance of Venus, it would seem that the great inclination of the equatorial plane was, (39), brought about by the attractive force of the Earth-mass of *greater density;* but, in the present instance, the like effect, as already shown, seems to have been due to proximity of *the great mass* of ♄; though, (3), the *density* of the existing planet Saturn, as exhibited in Table (A), is the *least* in the whole planetary system.

But even *that* is here found to be *a fact in place.* For the drawing over, (41), of a mass nearly equal to $1\frac{4}{10}$ of that of Uranus, from a region in which the *mean density* of the nebulous material was far inferior to that of the ♄-mass,[1] could hardly fail to have resulted in a mean density of the existing Saturn, such as we find.

5.

The scrupulously exact coincidence of the numbers in the column of *Law* with those in the column of *Fact* in Table (B), in (14), approaches the nearest to an exception, in the very instance of Uranus; the existing Uranus being 0.374 of the Earth's distance *within* the distance due to Uranus in accordance with *Law 2d*, in (10); though even that difference is less than $\frac{1}{80}$th of the whole distance of Uranus itself. But this, if we give it any weight at all, is, again, *a fact in place.* Uranus in the drawing over of the material towards ♄, may, perhaps, have somewhat *fallen in.*

6.

The acquisition of so much additional material, drawn in from a great distance, must, it would seem, have the effect of giving to the condensing Saturn-mass a much more oblate form than that which would otherwise have pertained to it; which seems to be confirmed by the fact that the outermost satellite is at the dis-

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[1] For the probable ratio of the densities here in question, see the paper of Mr. Trowbridge already referred to in the *Note* to (38).

tance of more than 64 radii of Saturn from his centre; while the distance of the outermost satellite of Jupiter, measured in the same way, is scarcely 27 radii of its primary.

And the *comparatively feeble light* of this same outermost satellite of Saturn is withal consistent with a *low density* of that satellite;[1] a fact also *in place*, in view of the acquisition of a less dense material from the planetary region *exterior* to the ancient Saturn ♄: the outermost satellite, in the view of the hypothesis as to its formation, being most probably constituted of the portion the least dense of all.

7.

Such being the special form and constitution of the Saturn-forming mass—the formation of the extensive system of satellites might have been nearly completed, *in advance of the "abandonment" of the material* which now constitutes *Saturn's rings;*[2] or that satellite formation, at least have gone so far, as to *keep the rings* in their *form* and *general arrangement*, while Saturn, condensing, shrank away from the rings, yet with his central position with regard to them (or rather their corresponding arrangement around him) preserved; *the conservative power* of the satellites, in these respects, being exerted in those very ancient times, even as now.[3]

It was then, it would seem, the drawing over and inward of the material which else had constituted the half-planet between Saturn and Uranus, that, as has been said, gave to Saturn and to his system the special form and arrangements that rendered the retaining of the rings *as* rings a possibility; which has made them an actuality; made Saturn what the author of the Novum Organum would term an "*instantia solitaris*," in the solar system.

8.

The same processes of the transference and combination of material here insisted upon, seem also to have affected the *inclination of Saturn's own equator*, and *that of almost the whole Saturnian System, to the plane of the planet's orbit.*

For this great planet's equator, and his rings, and the orbits of his satellites[4] are inclined at an angle of more than 28° with the plane of his orbit; while the inclination of Jupiter's equator, and that of the orbits of three of his satellites, does not much differ from 3°.

9.

Another relation may possibly have some significance in this connexion; viz., the ratio of the *periodic time* of the interior half-planet ♅i to the periodic time of the ancient Saturn ♄.

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[1] Not that the phenomenon of a comparatively feeble light would absolutely *require* the supposition of a low density; but, as stated, the one thing would be consistent with the other.

[2] There being material for that so *far outward* in the direction of the plane of the equator of the *very* oblate spheroid, or near to that; the spheroid being made so very oblate *by the acquisition from without of the material of* ♅i.

[3] For "no planet can have a ring, unless it is surrounded by a sufficient number of properly-arranged satellites. *Saturn* seems to be the only planet which is in this category; and it is the only one, therefore, which could sustain a ring."—Prof. Peirce, On the Constitution of Saturn's Ring, in the *Astronomical Journal* No 27, p. 18.

[4] All but that of the outer one.

For the mean distance from the sun of the (now missing) interior half-planet ⛢*i*, and that of Saturn [as recorded in the column of Law in Table (B), in (14)] being, respectively, 14.64275 and 9.44511, the application of Kepler's 3d Law will give us the corresponding periodic times; and then the measurement of the greater of these by the less, will show the periodic time due to the half-planet ⛢*i* to be to the periodic time of the ancient Saturn ♄ at its theoretical distance, in the ratio of 1.9303 to 1; and a still more scrupulous determination of the data in question than that exhibited in Table (B), might, perhaps, show the ratio to be very accurately that of 2 to 1.[1]

But with this ratio existing, the perturbations of one of the masses by the other at their nearest approach (intensified, it may be, by eccentricity of form or of orbit; or otherwise) would *recur* after *every two* subsequent revolutions of the ancient Saturn ♄; and very possibly the effect of those perturbations become, in this way, *cumulative;* and thus the passing over of the material of the half-planet have been furthered and aided, until its mass was absorbed by the ancient Saturn ♄.[2]

10.

It is not inconsistent with all that has just now been stated, that the term for the distance of Saturn reported in the column of *Law* in Table (B) is *less* than the corresponding term in the column of *Fact;* the ancient Saturn ♄ having, as it were, been *drawn outward* in the completion of the catastrophe of the absorption of ⛢*i;* while Uranus, as indicated in Consistency 5 of this series, may, perhaps, have somewhat *fallen in.*

11.

The (additional) 11th of these consistencies has much more extensive relations; some of which will here be exhibited and explained; they being especially such as are comprehended under the following title:—

The more Ancient Arrangement of the Material of the Planetary System.

For if—*always adhering to the hypothesis that the material of the existing Saturn was increased in the way so often already specified*—we endeavor to show what was *the more ancient combination and arrangement of the material of the solar system* (viz., ere the rending and the rupture, of which we now seem to find traces, were, in all their extent, accomplished), we shall find that, by regarding the masses in question (*half-planets, Asteroid mass or masses,* etc.), as recombined about their respective centres of gyration, and then ascertaining the *positions* of those centres, to serve as our points of reference, we shall thus obtain *a new and fully justified series of terms,* in which, very much as in the other instances of leading ratios in the planetary, and also in the satellite systems, every term will have a ratio to the next

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[1] The distance of ⛢*i* being, as stated, 14.64275; then, to perfectly justify a ratio of the periodic times of 2 to 1, would require the distance of the ancient Saturn ♄ to be 9.24562 instead of 9.44511.

[2] [For a further discussion and application of what is here intimated; as well as that of what more the relation in question may be significant, see *Articles* (64) to (67) inclusive.]

succeeding term, which will, here, *decrease* very slowly, but regularly, in the progress inward.

(44) With respect, then, to this recombination—

The value of the 1st, or *Neptune-term* of the series, closely corresponds to that in Table (B) of the completed arrangement of the Planetary System in (14).

For the *2d term* of the series—

Double Planet Arrangement.

Whole planet mass (U). (*a*). The mass of Saturn being reduced to that of ♄—to furnish the material for the half-planet ♅*i*—that half-planet must then be regarded as being restored to its appropriate place [as the same is exhibited in Table (B)].

(*b*). The two half-planets, Uranus and ♅*i*, must then be regarded as combined around their centre of gyration to form the *whole-planet mass* (U).

Whole planet mass ♄. The mass of ♄ will then be left at a *whole-planet distance.*

Then, (*c*).—The *whole planet mass* (U), accumulated anew (as already indicated), must be combined with the mass ♄ to form from *both*, around *their* centre of gyration, a quasi *double-planet* mass [(U) ♄]; to furnish the *2d term* required.

Jupiter will itself, in its mean distance from the sun, furnish the 3*d term.*

Mars and the *Asteroid mass* (A) will, in the quasi *double-planet* arrangement, at their centre of gyration, furnish the 4*th term ;* designated as that of [♂ (A)].[1]

The *Earth* and *Venus*, now existing as separate *half-planets*, will, in a *whole-planet* arrangement, furnish (at their *centre of gyration*) the 5*th term* very near, (39), to the already recognized limit (⊕♀). This 5*th term* is then designated as that of [⊕♀].

Mercury, in its mean distance from the sun, furnishes the 6*th term.*[2]

[1] In the computation of this 4th term, such a value has, of necessity, been attributed to the asteroid-mass as would make that 4th term in the column of Fact, absolutely the same with the corresponding term in the column of Law. But the value of the asteroid-mass thus determined, is confirmed in a way which cannot but be regarded as extraordinary. [See *Article* (46).]

[2] Neither the *aphelion* nor the *perihelion* distance appearing; though the one is found at a *whole-planet distance*, and the other at an *exterior half-planet distance*, in Table (B), in (14). Mercury, then, at a distance the mean of these two (but in another arrangement) has thus characteristics approaching to those of *a double-planet* [as was intimated, though not explained in (9)]; and this with an appropriate place in the series in which the *double-planet* arrangement appears; the difference between this and the otherwise analogous terms of the arrangement being, that whereas, in the other cases, the material of the two planetary bodies (with reference to its more ancient state) is regarded as accumulated anew, and, as it were, in some measure, reconstructed about the centre of gyration of those bodies; the actual combination, in an analogous position, seems to be found in the *existing planet*, Mercury itself.

The conditions prevalent in this series (with a quasi double-planet arrangement for every *alternate* term), require that the mean ratio R_1 should nearly $= r^{\frac{3}{2}}$, r being the mean leading ratio for the *whole-planet* arrangement in Table (B), in (14).[1] Accordingly we find that, with the mean value of r, in Table (B), [which, (13), $= 1.8253$], that $r^{\frac{3}{2}} = 2.4660+$, while the mean value of R_1 prevalent in this new series, is 2.4021

(45) The whole arrangement, in accordance with what has now been stated, is exhibited in the following table; the symbols of mode of connexion, and dependence, etc., being similar to those in Table (B), in (14).

TABLE (F).

More Ancient State and Arrangements of the Planetary System.

NAMES, etc.	SYMBOLS.	LAW.	FACT AND DERIVATIONS.	Diff. L.—F.	Diff. in terms of quantity measured.
NEPTUNE	Ψ	30.06039	30.05733	+0.003+	+0.000+
½ *planet Uranus* } ½ *planet* ♅*i* } Whole-planet (U) } Whole-planet ♄ } ..	[U♄]	12.44376	12.40099	+0.043	+0.003
JUPITER	♃	5.16574	5.20280	—0.037	—0.007
Asteroid mass (A) } Mars .... } ..	[♂(A)]	(2.15051)	(2.15051)	......	......
Earth....... } *Venus*....... }	[⊕♀]	0.89780½	0.88665	+0.011	+0.013
MERCURY............	☿	0.37589	0.38710	—0.011	—0.030

The values of the ratio R_1, which determine the numbers in the column of *Law*, are—

		Diff.	
Ψ to [(U)♄]	2.4157		
		0.0068	
[(U)♄] to ♃	2.4089		
		0.0068	
♃ to [♂(A)]	2.4021		Mean 2.4021.
		0.0068	
[♂(A)] to [⊕♀]	2.3953		
		0.0068	
[⊕♀] to ☿	2.3885		

The mean value of R_1 is, then, very nearly 2.4, which $= \frac{24}{10} = \frac{12}{5}$, so that every

[1] It being among those conditions that the *centre of gyration* of the component masses should very closely correspond in its position with that due to the intermediate term in the quasi *double-planet series;* a fact which itself seems to indicate, that the law of apportionment of the masses is not independent of that of the distances, but that the one (in the mathematical sense of the term) is a *function* of the other.

term, after the first, is $\frac{5}{12}\pm$ of that which immediately precedes it; instead of $\frac{5}{9}\mp$, which is the whole planet ratio in the existing planetary system.[1]

Now, it is especially to be again observed, that the *2d term* of the series in this Table, in the way in which it is here obtained, *supposes*, and it *depends* upon the supposition, that the material of the missing half-planet ♁*i passed over and was combined with* the other portion of the Saturn-forming mass, to, thus, construct the existing planet Saturn; and it is, (44), by supposing that process *reversed—restoring* ♁*i to its place*—and then *combining* in the way already indicated, (44), that the *2d term* of the Table is obtained for the column of Fact, and can, consistently and accurately, occupy its place in the series;[2] so that *this 11th consistency*, supporting the hypothesis of the disappearance of the missing planet, in consequence of its mass having been drawn inward and combined with the Saturn-forming mass, has even more extended relations than the others.

Having, then, as far as may be, answered the question, (41), *What has become of the missing mass*, it may next be well to consider what more we may be taught by certain other relations exhibited in Table (F).

Mass of the Asteroids.

(46) With the term [♄(A)], [at the centre of gyration of Mars and the Asteroid mass (A), as found in Table (F), in (45)], and also with the mass of Mars taken as unity, and the mean distances, from the sun, of Mars and (A), respectively, in Table (B), in (14), we may determine m', the Asteroid-mass which will be required to justify the term [♄(A)] in Table (F); the case being similar to that of the *interior* half-planet ♁*i* in (41); except that the value of m', the *exterior* mass, is here required instead of m.

Substituting in the equation, in (41), the values here indicated, we shall find m', *the Asteroid-mass*, = 0.58929 of *the mass* of Mars.

This, with the mass of Mars, as in Table (A), in (3), [$= \frac{1}{3200900}$], will make *the mass of the asteroids* = $\frac{1}{5431814}$ of the mass of the sun.

(47) Now M. Le Verrier, in the *Comptes Rendus*, tome lxv, p. 880 (Nov. 25,

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[1] As R_1 here approximates to $r^{\frac{3}{2}}$ [r being the ratio for the whole-planet terms in Table (B)], R_1 will also, incidentally, express very nearly the ratio of the *periodic times* due to the whole-planet distances. Accordingly we find that the ratio of the periodic time of Saturn to that of Jupiter = 2.4697; while the nearly corresponding value of R_1, as stated in (45), is, as near as may be, 2.4089.

[2] Not only so, but if *leaving out the hypothesis here in question*, we attempt to form the 2d term of the series with the Saturn-mass as it exists, we shall, of course, *fail;* since the placing of so large a portion of the same masses so much *farther inward*, will, at once, displace the centre of gyration in the same direction, and so make the term too small. And the same effect would even be manifest, if we might suppose a group of asteroids to exist in this region; but that, (42), is inadmissible.

On the first of these two suppositions, the centre of gyration would be displaced quite the whole of the Earth's distance from the Sun [being at 11.35 instead of 12.40]; and if the second supposition were admissible, the displacement would be nearly ½ that distance [being at 11.96 instead of 12.40].

1867) has given us the following equation, dependent on the necessity of an admitted increase in the motion of the perihelion of Mars.

He states that, in so far as we now know—

Ten times the correction of the mass of the Earth, *plus* three times the mass of the small planets, in a mean distance reference of the group, would make a sum equal to 1.38; the mass of the Earth deduced from the parallax of Encke, 8″.58, being taken for unity.[1] This mass is $\frac{1}{354936}$.

The mass of Mars which M. Le Verrier employed in his investigations, would seem to be the same with that which he has, provisionally, attributed to that planet in the *Comptes Rendus* for July 22, 1872; viz, 0.000000333 of the sun.

With these values of the data, the equation of M. Le Verrier will give us, FOR THE ASTEROID MASS, THE SAME FRACTION OF THE MASS OF MARS WITH THAT WHICH JUSTIFIES THE TERM [♄(A)] IN OUR TABLE (F); *if we make* THE SOLAR PARALLAX 8″.896;[2] which is a value included within the present limits of uncertainty, *and near to the mean of all the more recent determinations.*

(48) If, then, fortified by these several coincidences, we allow any weight to the determination of the value of the *Asteroid mass* derived from the justification of the term [♄(A)] of the series here in question; it may be noted that this value, (41), depends on the ratio of the difference of the squares of the terms [♄(A)] and Mars to the difference of the squares of (A) and [♄(A)]; and the tabular values of the quantities represented in the terms thus involved, may all be considered as being approximately well-determined.

[It will, moreover, be observed that the several independent elements which have entered into the computation of this result are:—

1. The leading ratio r, in Table (B), in (14).
2. The leading ratio R_1, in Table (F), in (45).
3. The application of the formula *for the centre of gyration;* and

---

[1] . . . . "on doit dire que dix fois la correction de la masse de la Terre, plus trois fois la masse de l'ensemble des petites planètes distribuées en moyenne, d'après ce qu'on en sait aujourd'hui, doit faire une somme égale à 1.38; l'unité étant la masse admise pour la Terre quand on la déduit de la parallaxe d'Encke, 8″.58."

[2] For, $\left(\frac{8''.896}{8''.58}\right)^2 = \frac{\textit{increased mass of Earth, M}}{1}$; the mass due to parallax 8″.58, being $=1$

M being thus determined—

Then $M - 1 =$ *increment* of Earth's mass $= i$.

Then m' being *asteroid mass*, M. Le Verrier's equation gives—

$$10i + 3m' = 1.38; \text{ whence}$$
$$3m' = 1.38 - 10i, \text{ and}$$
$$\textit{asteroid mass, } m' = \frac{1.38 - 10i}{3};$$ the mass of the Earth due to parallax 8″.58 being 1.

Then $\frac{1}{354936} m' =$ *asteroid mass* m'' in terms of the *Sun's mass* 1.

And *this* last value is *our* fraction (0.58929) of *M. Le Verrier's mass of Mars, i.e.* the *same* fraction of the mass of Mars (taken = 1), which *justifies the value of our* [♄(A)] *term* in our Table (F).

4. The mass of Mars itself, deduced from the mutual action of it and those of the other planets.]

But the value of the same *Asteroid-mass*, as derived from M. Le Verrier's equation, depends on $\frac{1}{3}$ of *ten times* the excess above 1 of $\left(\frac{8''.896}{8''.58}\right)^3$. This value, then, albeit that it wholly depends on ascertained facts for its data, is, nevertheless, very *sensitive* to any, the smallest, change in the value of the solar parallax.

[In a subsequent Memoir on the Masses of the Planets and the Parallax of the Sun, in the *Comptes Rendus*, for July 22, 1872, M. Le Verrier, as the result of a discussion of the *secular variations* of the elements of the orbits of *Mercury*, *Venus*, the *Earth*, *Mars*, and *Jupiter*, states that it is probable that the attraction of the minor planets amounts, up to the present time to a quantity which may be neglected.[1]]

(49) The value of the Asteroid-mass, which we have thus obtained, is, as far as may be, confirmed by yet another consistency.

For with this value of the mass, at distance (A) in the column of *Law in* Table (B), and other masses and distances in Tables (A) and (B), [(3) and (14)], we shall find that the neutral point, or point of equal attraction of this same mass, is, on the side of Jupiter, at the distance 3.16559 from the sun. And the similar limit, on the side of Mars, is at the distance from the sun = 2.13869.

These numbers at once suggest the limits (thus far recognized) of the mean distances of the asteroids.

The supposition of a *half-planet* arrangement of the material in the progress of its early "abandonment" will, however, better provide for all this; as well as exhibit yet other consistencies, as will be shown hereafter.[2]

Peculiar Relations of the Planet Mercury.

(50) From Table (B) in (14) and Table (F) in (45), we find that the position and relations of Mercury may be represented as follows:—

	Table (B).	*Table* (F).	
Limit or term	(⊕♀)	.... [⊕♀]	$R_1 = r^{\frac{3}{4}}$
Whole planet ratio, r	Aphelion of Mercury		
$\frac{1}{2}$ planet ratic, $r^{\frac{3}{4}}$	*Perihelion of Mercury*	...(at mean dist.) MERCURY	

so that Mercury, when *in aphelion*, is in the position due to a *whole-planet;* and when *in perihelion* his distance is that due to a *half planet.*

[1] As quoted in the translation of W. T Lynn, B. A , in the *Monthly Notices of the Proceedings of the Royal Astronomical Society,* vol xxxii., No. 9, p. 323.

[2] See *Articles* (60) and (108).

Then, at his *mean distance* (half-way between the two) his place is that of an *almost double-planet*, in the special arrangement in Table (F).

Of these it may be said:—

1.

That these several peculiarities seem, at once, *to be reconciled and explained* by the supposition that the condensing material (ring, or shell, etc.) which was in position to have formed a whole planet at the *aphelion distance*, and another portion of the condensing material (ring, or shell, etc.) which was in position to have formed what we have termed an *exterior* half-planet, at the *perihelion distance*, have been *combined* to form the existing planet; which, thus, is made up of a *whole-planet mass* and a *half-planet mass.*

2.

But all this *accounts for* and *explains in mode and in measure*, the VERY GREAT ECCENTRICITY OF THE ORBIT OF MERCURY; his perihelion distance not *extending beyond* the centre (or a point near the centre) of gyration of the half-planet mass (ring, or shell, etc.) due there; and his aphelion distance, *reaching out* to the centre of gyration, or near it, of the whole planet mass due *there.*

Mass and Distance of a possible Planet interior to Mercury.

(51) The position of the perihelion of Mercury has, (14), been shown to be that due to an *exterior* half-planet. Hence the distance from the sun of the next planet interior to Mercury may, most probably, be ascertained by dividing the term value of Mercury's perihelion distance, in the colum of *Law* in Table (B), in (14), by the value of $r^{\frac{1}{2}}$, in accordance with *Law 3d*, in (10).

The value of $r^{\frac{1}{2}}$, for this region of the system, is 1.3733.

Performing then the division thus indicated, we shall have the distance from the sun of the planet interior to Mercury—

$$\text{☿}_i = 0.20836.$$[1]

We may also ascertain the *whole-planet* position next to that due to the *aphelion* of Mercury, by dividing the aphelion term in the column of Law in Table (B), in (14), by the value of r, in accordance with *Law 1st* in (10).

The value of r, for this region of the system, is 1.8736. Dividing the value of the *aphelion limit* by that number, will give for the *whole-planet limit* interior to Mercury's aphelion distance, the value 0.24422 + .

Thus, then, we shall have the following arrangement:—

$r^{\frac{3}{4}}$	. . . . . (Whole planet limit) aph. distance		0.45758	r
	(*Exterior* ½ *planet-limit*) *per. distance* . .	0.28573		
$r^{\frac{1}{2}}$	. . . . . whole planet limit		0.24422	
	Interior half-planet ☿*i*	0.20836		

[1] This is very accurately the distance required (by *Kepler's 3d Law*) to justify the periodic time of the so-called "planet Vulcan," as the same has recently been ascertained by Prof. Kirkwood, on the hypothesis, that the appearances of certain solar spots were due to the transits of such a body.

Then for the *mass* of the *interior* half-planet ☿*i*, we need first to redistribute the material of Mercury, so as to place its *whole-planet portion* at the aphelion, and its *half-planet portion* at the perihelion; to come back to the forming state, etc., described and exhibited in symbol in (50).

Putting then the whole mass of Mercury = to 1; if *that* be so distributed to the aphelion and perihelion positions, that *the centre* of *gyration* of the distributed portions shall be found at Mercury's *mean distance*,[1] we shall have—

0.5617245 of *Mercury's mass*, for *the aphelion*, and
0.4382755 " " " " " *perihelion*.

The values *thus* far requisite having been ascertained, the case is but a repetition of that of the *mass* of ♁*i* in (41); and by substituting the values now before us, and reducing, we shall find the value of the *mass* of the *interior* half-planet—

m of ☿*i*, interior to Mercury, = 0.594059 of *the mass* of *Mercury*.

(52) Now M. Le Verrier, in the *Comptes Rendus*, tome XLIX. p. 382, (Sept. 1859), speaking of a cause adequate to produce an ascertained secular motion of 38″ in the perihelion of Mercury, admits the supposition of a hypothetical planet, situated between Mercury and the Sun, and says that, as the hypothetical planet ought to impress on the perihelion of Mercury a secular motion of 38 seconds, the resulting relation between its (the planet's) mass and its distance from the sun will be such that, in measure, as we suppose the distance less, the mass will be increased, and the converse: and he adds, that, "For a distance a little less than the half of the mean distance of Mercury from the Sun, the mass sought would be equal to that of Mercury."

The mass which, *on our own plan*, *in the following out of our own hypothesis*, (51), we have found for the hypothetical planet is 0.594059 *of the mass of Mercury;* and when, in conjunction with Mercury, as seen from the sun, the distance between the two planets [see (51) and Table (A), in (3)], would be

$$0.38710 - 0.20836 = 0.17874;$$

and "a mass equal to that of Mercury," *similarly situated*, would have the same attractive force with that due to our hypothetical planet, at a distance, for that mass, inside of Mercury = to 0.23190, *i. e.*, a distance from the sun = 0.15520; which is indeed, assuredly, somewhat "less than the half of the mean distance of Mercury from the Sun," which ½ distance, accurately, = 0.19355.

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[1] For this purpose, $m+m'$, the sum of the two masses, being put = to 1; $m'=1-m$.

Also—since the ratios of the distances are known, or may be readily ascertained—if (C) be the distance of the centre of gyration, and the distance of the outer body = q (C), and that of the inner = p (C); then, substituting in *Eq.* (C) in (17), and reducing, we shall have, for the fraction of the whole mass pertaining to the inner body,

$$m = \frac{q^2 - 1}{q^2 - p^2};$$

which will, also by substitution and subtraction, give us m', since it = $1 - m$.

All this, *so* far, approximates to an accordance with M. Le Verrier's required action of the mass in question. It is then sufficiently manifest that our hypothetical planet, as to *mass and distance both*, would be such as measurably to satisfy the conditions of the ascertained perturbation; and so we need not pursue the investigation of a troublesome problem any farther.

Peculiar Relations of the Living Force of (simultaneous) Rotation of some of the Planetary and Satellite Masses.

(53) If Jupiter and Saturn should (or if they did) turn around the sun, *in the same time;* the moment of rotation must, in the instance of either, be represented by the formula, *mass* × (*velocity*)^2; or, as velocity in this case would be, as a, the radius vector of rotation, the *ratio* of the moments will be obtained by comparing *mass* × (*radius vector*)^2 of the one with *mass* × (*radius vector*)^2 of the other. So with m and m', respectively, for the masses, and a and a' for the radii vectores; *i. e.* the mean distances from the sun, as in the column of *Law in Table* (*B*), in (14), and the masses, as in *Table* (*A*), in (3); we have—

For *Jupiter*, $ma^2 = 0.026142$.
For *Saturn*, $m'a'^2 = 0.025477$.

or with the distances as in column of *Fact* in Table (B); we have—

For *Jupiter*, $ma^2 = 0.025832$.
For *Saturn*, $m'a'^2 = 0.025985$.

The approach to a ratio of equality is here very close.[1]

There is also an approximation to the same state of things in the following cases.[2]

The respective moments of (simultaneous) rotation of ♄̂ (*i. e.* Saturn *reduced* to its *ancient state*), of *Uranus*, and also of ♅*i* [the half-planet (supplied) *interior* to Uranus], are all nearly equal to one another; the ratios being—

$$\frac{m\,r^2 \hat{♄}}{m'\,r'^2 ♅} = 1.1431 \;.\;.\;.\;.\;.\; (1).$$

$$\frac{m'\,r'^2 ♅}{m''r''^2 ♅ i} = 1.0060 \;.\;.\;.\;.\;.\; (2).$$

Then, when the *combined* masses of Saturn and Uranus [in the *More Ancient State*, as exhibited in the *term* [(U)♄̂], in *Table* (F), in (45)], are compared with *Neptune* in respect to the moment of (simultaneous) rotation; we have for the ratio—

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[1] This curious relation was first made known by the author of this paper to the *American Association for the Advancement of Science*, at their Meeting in Montreal, in 1857; also the division into shells, etc.

[2] Which might be somewhat varied, were all the masses more accurately determined.

$$\frac{m_1^2 r_1^2 \text{ of } [(\text{U})♄]}{m'''^2 r'''^2 \text{ of } ♆} = 1.1101 \; . \; . \; . \; . \; . \; (3)$$[1]

Lastly, in the *System of Saturn*, m being the mass of the outer, and m' that of the inner bright system of rings; we shall have for the ratio of the moments of (simultaneous) rotation—

$$\frac{m \times a^2 \text{ of } \textit{outer rings}}{m' \times a'^2 \text{ of } \textit{inner rings}} = 1.1400 \; . \; . \; . \; . \; . \; (4);$$

the rings being respectively referred, each to its centre of gyration [obtained as in (16)].

[Then, since the rings in Table (C) in (18), have their places as *satellites;* if the *periodic times* of the rings referred to their centres of gyration agree with *Kepler's 3d Law*, and so actual velocities are as $a^{\frac{1}{2}}$ to $a'^{\frac{1}{2}}$, and hence their 2d powers as a to a'; we shall have for the ratio of the moments of rotation of the existing and turning rings

$$\frac{m' \times a' \text{ of } \textit{inner rings}}{m \times a \text{ of } \textit{outer rings}} = 1.0752.\Big]$$

There is a very close resemblance between ratios (1) and (4).[2] Were, then, those *ancient masses* compared in (1), *ring-like in form;* and *did* the masses, with nearly equal moments of (*simultaneous*) rotation, go round the central body together?

If, in an ancient state, they were *parts of the atmospheres of their primary and central body*, in every case; then they *did* go round *together*. But, whether we admit any part of that hypothesis, or else reject any portion, or all of it; THE RATIOS REMAIN, and seemingly without that hypothesis, they remain unaccounted for.

There is yet another aspect of the matter, and that is—that the rings or shells, etc., separated about the time *when the moments in question became nearly equal.*

Application of other Conditions appertaining to the ring-like Form. What succeeded these.—Position of great Planets, and of largest Satellites.

(54) It has, (16), been shown that the centre of gyration of a *homogeneous* ring is in the circumference in which the mass of the ring is bisected; and that thus, we have

$$(C)^2 = \tfrac{1}{2}(R^2 + r^2)$$

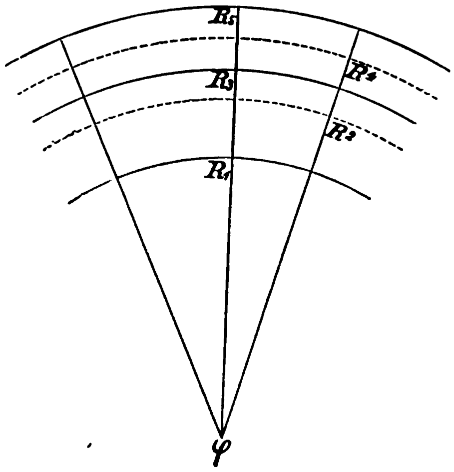
---

[1] Ratios (2) and (3) are consistent with the supposition in (43), that the material of Saturn was gathered in part from *the interior half-planet*, now missing (the values ♄ and ♁*i* being dependent on that); but they did not seem to be of such importance as to require their admission as *Coincidence 12th* of the series exhibited in (43) and (45).

[2] Though it should not be overlooked that ratio (4) is that existing in a satellite system, which is here compared with those found in the system of the primary planets.

(C) representing ϕR_3 in the figure, *i.e.* the distance of the centre of gyration from the centre of force, and R and r, respectively, the radii of the edges of the ring, so that we have

Fig. 11.



$$\overline{\phi R_3}^2 = \tfrac{1}{2}(\overline{\phi R_1}^2 + \overline{\phi R_5}^2).$$

Now the like being also true of the half-rings, with *their* centres of gyration at R_2 and R_4, respectively; we shall also have

$$\overline{\phi R_2}^2 = \tfrac{1}{2}(\overline{\phi R_1}^2 + \overline{\phi R_3}^2); \text{ and}$$

$$\overline{\phi R_4}^2 = \tfrac{1}{2}(\overline{\phi R_3}^2 + \overline{\phi R_5}^2);$$

from which, by substitution and reduction, we shall obtain

$$\overline{\phi R_3}^2 = \tfrac{1}{2}(\overline{\phi R_2}^2 + \overline{\phi R_4}^2);$$

in which the *centres of gyration of the half-rings* respectively, take the places of the *edges* of the whole ring.

(55) The supposition here throughout has been that all the material was homogeneous. But as the "abandoned" rings, or ring-like masses, would increase in density inward, the centre of gyration for each half-ring, as well as that of the whole ring, would also, therefore, be *within* that assigned by the formula.

Nevertheless it would seem that this would affect, or rather has affected, the several quantities, proportionally.

Accordingly, we find that *the mass* of the system of the inner bright rings of Saturn *is* considerably *greater* than *the mass* of the system of the outer bright rings; yet the other condition here in question is fulfilled.

For the centre of gyration of the outer bright rings, [Table (C) in (18)], is at the distance	2.1165.
And the centre of gyration of *both systems* of the bright rings, as obtained independently by the general formula, is at distance .	1.9090.
And that of the system of the *inner* bright rings is at . . .	1.7097.
Now the sum of the squares of the first and last of these numbers is	7.16399197;
and $\frac{1}{2}$ of the same =	3.58199593+
And the square of the intermediate number, 1.9090, =	3.64428100;

showing a very close correspondence with the formula.

Accepting, then, this result as *an induction*, we shall find, on trial, in the same way, a semblance of a *ring-like* form of the "abandoned" masses, apparent, even in the case of the *Earth* and *Venus*.

For the sum of the squares of their mean distances [as those distances are given in the column of *Law* in Table (B) in (14)] is . . .	1.51928
and $\frac{1}{2}$ sum =	0.75964
And, (C) being distance of the centre of gyration, . . . (C)^2 =	0.78616;

in which case $(C)^2$ is the greater because of the superior density of the Earth. [And the great relative distance of our own satellite (nearly 60 radii of the Earth) as, in the similar instance in Saturn's system, is also [6 of (43)] indicative of a great oblateness of the nebulous material at some stage of its progress.]

(56) Again, a like relation is found in the case of the mean distance and centre of (simultaneous) gyration of *Uranus* and *Neptune*.

In the instance of these we have an approximation to *equality in the masses*;[1] the ratio of the mass of Neptune to that of Uranus being

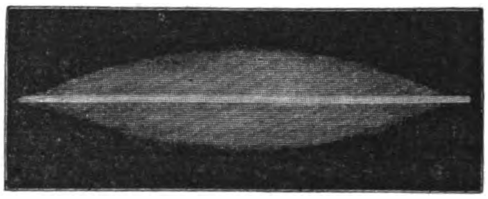
$$\frac{m\Psi}{m'\text{♅}} = 1.11678.$$

Moreover (C), the centre of gyration of the two planets is at the distance 25.4457–; and while

$$\tfrac{1}{2}(\textit{mean dist. } \Psi)^2 + \tfrac{1}{2}(\textit{mean dist. } \text{♅})^2 = 635.704$$
$$(C)^2 = (25.4457-)^2 \quad . \quad . \quad . \quad . \quad . \quad = 647.481$$

This is consistent with a *ring-like* form of the two masses in question, after the "abandonment" of the material of which they were constituted; the flowing over of material in this outer portion of the oblate solar atmosphere having given to the whole, or, at least, to both the parts of the masses in question, a form not unlike that of a thick ring.

Fig. 12.



All this is consistent with that form, yet does not *require* the masses to have had such a form; since, (17), the equation here in question would, accurately, exist in the case of *any equal masses*.

(57) The state of things arrived at (perhaps later) in the case of *Jupiter* and *Saturn*, (53), seems to be inconsistent with a mere ring-like form for both masses; but to be a consequence of the accession of material from regions of the sun's atmosphere extra-equatorial. Accordingly we shall find that the equation here in question does not obtain in that instance.

But under the conditions approximated to in the case of planets exterior to them, and at length attained in the instance of those two great masses, viz.

$$ma^2 = m'a'^2,$$

we have the masses inversely as the squares of the radii of gyration; so that the resulting planets must increase in mass, in the progress inward, until we come to the instance of Jupiter, the greatest of all;[2] the ring-like masses, or the shells, though successively decreasing in volume, yet increasing more rapidly in density,

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[1] The mass of Neptune is the greater; Uranus having just possibly lost somewhat in the process, (43), which carried away the mass of the now missing planet.

[2] Mr. Trowbridge, in his investigation already referred to (*Note* to 38), [in 1864], shows that this would be true of the "abandoned" rings. But the increase of the mass of the great planets, in the progress inward, would seem to be too rapid to be explained by *that alone*. The other changes and relations in question may, as it would seem, have been even more efficient; and the *most* of these were indicated by the author of this paper in 1857, as heretofore stated in the *same Note to Article* (38).

for some distance within; so that the planets of greatest mass would *not* be the *outermost*, but the masses of the successive planets will be greater and greater, so long as the density increases in a greater ratio than that in which the volume diminishes; aided, withal, by the whole-planet arrangement, which supervenes in the Saturn and Jupiter arrangement, and, in the instance of Saturn, (42), by the half-planet acquired.

And this arrangement of the masses we actually find, with some variation in the instance of Uranus.[1]

(58) Closely analogous to this arrangement of the masses in the great planetary system is that which we find in the *System of Saturn;* viz. Japetus outside, for one of the larger satellites, followed by Titan, the Jupiter of the system, with smaller satellites after it (Hyperion *before* it, in the place analogous to that of Uranus), and other satellites, larger than Hyperion, farther inward.

(59) Then too, in the *System of Jupiter*, the relative masses of the satellites are—

Satellite IV.	42659
" III.	88497
" II.	23235
" I.	17328

so that the mass of Satellite IV. approaches to being more than double that of either Satellite II. or Satellite I.; while the mass of Satellite III. is more than the double of *that* again; the *great* masses *outside* of the others; and yet, as in the other systems, the *greatest of all* not the *outermost.*

Arrangements of the Asteroid-mass.

(60) The neutral points for the Asteroid-mass, towards *Jupiter* on the one side and *Mars* on the other, have, (49), been already stated. But when we come to apply the formula for the ring-like mass; viz. that which has, (55), been especially in question, we do not succeed. We thus have a *negative* indication that the Asteroid-mass, *as a whole*, did not have a ring-like form.

But if we suppose a *half-planet* arrangement of the mass, we shall have

Distance of *exterior* half-planet	3.34083
" " *interior* "	2.47748
And then the sum of their squares	17.29905+
½ sum	8.64953—
Square of mean distance (A), in Table (B) in (14),	8.28067;

again approximating to the requirements of the formula.

The neutral point, or point of equal attraction, between *Jupiter* and the exterior half-planet will be	3.35790
That between the two half-planets,	2.94068
Between the interior half-planet and *Mars*,	2.14438

[1] May be in a measure accounted for and explained by the special influences to which, (43), that planet appears to have been subjected.

The first and last of these, toward one limit and the other, also indicate the *range* of the mean distances of the asteroids better than the result in (49). [The middle limit 2.94068 here given, is a little outside of the centre of gyration of the two half-planet masses, which is at whole-planet distance (A) of Table B, =2.87831 —the more dense material being inward: a state of things of which there is a distinct semblance, (19), in the previous example of *Saturn's rings*. In the case of the *Earth* and *Venus*, (39), the centre of gyration is without the neutral point, as it ought to be, because of the superior density of the earth.]

The exterior limit, 3.35790, at which the attraction of the outer mass and that of *Jupiter* would seem to have been in equilibrio, is scarcely 0.017 (of the Earth's mean distance) outside of the position due to the exterior half-planet.[1]

(61) The distances 3.34083 and 2.47748, respectively due to the exterior and interior half-planets, themselves exhibit approximations to the *aphelion* and the *perihelion distances of several of the existing asteriods ;* insomuch that their case in that respect resembles that of MERCURY, already commented on in (50): with the marked difference, however, that while the orbit of MERCURY is, indeed, limited in its aphelion by a whole-planet distance, and in its perihelion by the succeeding half-planet distance, the existing planet seems to have *combined in itself* the material which would have appertained to both the whole and the half-planet.

(62) The very small mass due to the exterior half-planet (0.4274 of the interior half-planet, or 0.2518 of *Mars*) would itself suggest the probability that but few asteroids were to be looked for at a mean distance, near to the outer limit 3.35790; and the progress of discovery, thus far, has justified such a conclusion.

Special Relations of the Moments of (simultaneous) Rotation (around the same centre) of the two supposed Asteroid-masses and that of Mars.

(63) The moments of (simultaneous) rotation of the *two* Asteroid-masses (*half-planetary* in position) and that of *Mars* have, respectively, the ratio of the following representative numbers:—

Exterior Asteroid-mass	2.8108	*Mean*, 2.4410
Interior " "	2.0712	
Mars		2.4679.

Of Missing Terms, or, at least, Varieties in Planetary or Satellite Series, other than those heretofore noticed; and the Explanation of the same.—A Resisting Medium.

(64) As "the comet of Lexell" had its orbit *twice* changed, as a special consequence of its periodic time being very nearly ½ that of *Jupiter*, so that the comet was for the second time brought very near to that disturbing planet after only two revolutions; so, also, it has been well argued that when the periodic time

[1] So that, as has often been surmised, the o'ermastering attraction of Jupiter must (it would seem) have interfered with the existence of the outer half-planet as such; and this, by an action not very unlike that of Saturn, (43), in preventing the continuance of anything like a half-planet interior to Uranus.

of the disturbing planet was very nearly a multiple of the periodic time of an "abandoned" ring; very similar effects would follow, which have, in part, at least, been indicated by Prof. Daniel Kirkwood in his paper *On the Nebular Hypothesis and the Approximate Commensurability of Planetary Periods*, in the *Monthly Notices of the Royal Astronomical Society*, vol. xxix. In that paper, at p. 99 of the volume quoted, he sums up, in part, what he had discussed, as follows:—

"A planetary particle at the distance 2.5—in the interval between *Thetis* and *Hestia*—would make precisely three revolutions while *Jupiter* completes one; coming always into conjunction with that planet in the same parts of its path.[1] Consequently its orbit would become more and more eccentric until the particle would unite with others, either interior or exterior, thus forming the nucleus of an asteroid. Even should the disturbed body not come in contact with other matter, the action of *Jupiter* would ultimately change its mean distance, and thus destroy the commensurability of the periodic times. In either case the primitive orbit of the particle would be left destitute of matter.[2] The same reasoning is, of course, applicable to other intervals;" and Prof. Kirkwood produces evidence to show that the "intervals in the asteroid zone"—however small at best—are yet appreciably greater in the instances of "nearly commensurable periods." With respect to the interval between the two *Rings* (or system of rings) of *Saturn*, Prof. Kirkwood, after a discussion of the distances and periodic times in question, concludes, "It is thus seen that the interval occupies precisely the space in which the periods of satellites would be commensurable with those of the four members of the system immediately exterior. As, therefore, the powerful attraction of *Jupiter* produces the observed *gaps* in the asteroid zone, so the *disturbing influence of Saturn's interior satellites is the physical cause of the permanent interval between the two bright rings*."

Prof. Kirkwood concludes his paper with the declaration that the Nebular Hypothesis . . . . . "assigns an obvious cause for the establishment of nuclei in such positions that their periods will be nearly commensurable with that of the disturbing body. As these nuclei would receive accretions of matter from portions of space both interior and exterior to their respective orbits, their distances from the central body, during their planetary growth, would not be liable to great variation."

(65) Now, with our half-planetary arrangement of the Asteroid-mass, (60), the periodic times of Jupiter, the *exterior* half-planet mass, the *interior* half-planet mass, and *Mars*, will, respectively, be related as follows; the coincidences, though not absolute, being yet very close—

P. Time (T) of *Jupiter* = 2 (T) of *exterior* asteroid-mass,
= 3 (T) of *interior* asteroid mass; and
(T) of *interior* asteroid-mass = 2 (T) of *Mars*.

Thus with the action of *Jupiter* on the one side, and *Mars* on the other, there would be abundant occasion for the effects under discussion.

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[1] All but the very distance of the *interior* asteroid-mass, as exhibited in (60).

[2] *See*, again, Consistency 9, in (44); referred in *Note* 2, on p. 30, to this place.

Then also, in view, (62), of the very small *exterior* half-planetary mass, in this instance, and the close approximation of *Jupiter's* o'ermastering influence; and the much larger, (62), *interior* half-planetary mass, and its special relations to *Mars* as here specified, we discern, at last, how the formation of half-planets in this region may have been prevented; also, why the range of the asteroids should be so extensive; why the eccentricity of their orbits should be so great; why so many have been discovered at distances approaching to that of the interior half-planetary mass, and even on the side toward *Mars;* and why so few have been found at distances approaching to that of the *exterior* half-planetary mass.[1]

Besides all this, we have the fact, that the actual distance of *Mars* [as seen in Table (B), in (14)], is appreciably *less* than the distance registered in the column of *Law; Mars*, like *Uranus* [*see* 5 of (43)], having seemingly *fallen in;* though not, like *Uranus*, influenced, to a proportionate extent, by a large planet interior to itself; yet the *acquisition* of sufficient material *from the interior half-planetary mass*, with the inferior velocity of revolution appertaining to that mass, would produce just such an effect.[2]

And the Earth-Venus mass, while it endured (if at all), would have had a periodic time $\frac{2}{5}$ths of that of *Mars;* and might, with the other influences in question, contribute to the very considerable *eccentricity* of the orbit of *Mars;*—on which, however, it does not seem to be justifiable to insist.

(66) In the *System of Saturn* there are withal vacuities, (64), in the series of satellites, under the conditions already specified in the other cases. Thus, in the large interval from Japetus to Titan, if the places for interpolated terms as indicated in Table (C), in (18), be compared with those which would be due to satellites with periodic times *commensurable* with the periodic time of Japetus, or with that of Titan, we shall have the following results:—

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[1] Then, among things *supposable*, but not as yet fortified by groups of coincidences, and which *cannot now be used in the way of induction*, are these: If either of the half-planets were after all formed, the oblateness of the nebulous material must have been so great that it might be questioned whether of the two possible forms of a rotating spheroid of equilibrium—the density and the time of rotation being given—the one usually differing but little from a sphere, the other, with the *equatorial diameter enormous* in comparison with *the axis*, the latter might not be the form of the spheroid here produced; it being such as the ring of *Saturn* might become if the body of the planet were removed, and the ring filled up so as to be imperforate. Such a form would be eminently unstable; and if it were broken up, the fragments would *all be small;* as the asteroids indeed are.

Then two such half-planets (with orbits, as has been seen, very eccentric) might all the more readily have realized the ingenious conjecture advanced by Prof. Vaughan at the meeting of the American Association for the Advancement of Science, in 1857; viz. that the asteroids were the fragments resulting from the *collision of two planetary bodies*, in that region of the solar system; thus presenting a new phase of the hypothesis of Olbers.

In the same category, as to not furnishing any induction as yet, may be included the fact that the orbit of Halley's (*retrograde*) comet very nearly (now) intersects that of Phocea.

[2] For additional proof of a half-planetary arrangement in the Asteroid region, see *Article* (108).

(Reckoning from Japetus inward), submultiples of periodic-time of JAPETUS, and corresponding distances.		Distances in accordance with ratios of terms in Table (C).	(Reckoning from Titan outward) multiples of the periodic-time of TITAN, and corresponding distances.	
P. TIME.	DISTANCE.		P. TIME.	DISTANCE.
$\frac{2}{3}$ that of JAPETUS	49.109	51.9925	$3\frac{1}{2}$ that of TITAN	51.037
$\frac{1}{2}$ " "	40.544	41.9986	$2\frac{1}{2}$ " "	40.782
$\frac{2}{5}$ " "	34.939	33.9271	2 " "	35.145
$\frac{3}{7}$ " "	27.919	27.4069 (*Hyperion*)	$1\frac{1}{2}$ " "	29.014

In the Interval from Titan to Rhea.

In accordance with Ratios of Terms in TABLE (C.)	(Reckoning from Titan inward) submultiples of the periodic-time of TITAN, and corresponding distances.	
DISTANCE.	PERIODIC TIME.	DISTANCE.
17.2598	$\frac{2}{3}$ that of TITAN	16.894
13.4556	$\frac{1}{2}$ "	13.947
10.8696	$\frac{1}{3}$ "	10.644
(*Rhea*) 9.5972	$\frac{2}{7}$ "	9.604

In this region the coincidences, it will be perceived, are more perfect than in the other region exterior to *Titan.*

But it is here, again, worthy of remark, that Hyperion, outside of Titan, in a place analogous to that of Uranus in the planetary system, has, like that planet, *seemingly fallen in somewhat* from its true position in series; as if influenced by the great interior body, under stringent circumstances. [*See*, again, 5 of (43).]

Exact Commensurability of Periodic Times.—Explanation of this.

(67) M. Laplace, in the course of his comments on his own hypothesis, especially notices and accounts for "the rigorous equality observed between the angular motions of rotation and revolution of every satellite;" all which will be considered in another connexion.

But, he adds, that "the first three satellites of Jupiter present a still more extraordinary phenomenon;" which consists in this, that "the mean longitude of the first *minus* three times that of the second, *plus* twice that of the third, is always equal to two right angles."

Next, with respect to the *existing* satellites of Saturn, we have the statement of Sir J. Herschel that "A remarkable relation subsists between the periodic times of the two interior satellites and those of the two next in order of distance, viz., that the period of the third (Tethys) is double that of the first (Mimas), and that of the fourth (Dione) double that of the second (Enceladus). The coincidence is exact in either case to about the 800th part of the larger period."[1]

Again, in the *American Journal of Science and Arts*, 3d *Series*, vol. iii, p. 67 (1872), is an extract from a letter of Prof. Benjamin Peirce to Prof. Newton, in which Prof. Peirce says: "I have discovered three fixed equations between the mean motions of the four outer planets. If the mean motions of Jupiter, Saturn, Uranus, and Neptune are respectively represented by n^{v}, n^{vi}, n^{vii}, and n^{viii}, these equations are—

$$4n^{\mathrm{vi}} + 9n^{\mathrm{viii}} = 16n^{\mathrm{vii}}$$
$$2n^{\mathrm{v}} + 17n^{\mathrm{vii}} + 6n^{\mathrm{viii}} = 12n^{\mathrm{vi}}$$
$$3n^{\mathrm{vii}} + 8n^{\mathrm{viii}} = n^{\mathrm{v}}$$

. . . . . To which he adds . . . . . "If all the three equations are admitted, the mean motions of three of these planets can be computed when the fourth is given;" and he exhibits the requisite equations. He states, moreover, that the reception of these "involves a laborious revision of the theory of these planets, ... and must seriously change the elements of their orbits."

Lastly;—to this, Prof. Daniel Kirkwood adds:[2] "The recent note of Prof. Peirce announcing his discovery of some interesting relations between the mean motions of the four outer planets, has recalled my attention to a number of similar coincidences detected by myself several years since, while engaged in a somewhat laborious examination of the planetary elements. Of these the following may be worth putting on record for future discussion:—

$$2n^{\mathrm{v}} - 3n^{\mathrm{vi}} - 11n^{\mathrm{viii}} = 0 \quad . . . . . (1).$$
$$2n^{\mathrm{vi}} - 21n^{\mathrm{vii}} + 30n^{\mathrm{viii}} = 0 \quad . . . . . (2).$$
$$3n^{\mathrm{v}} - 8n^{\mathrm{vi}} - 2n^{\mathrm{vii}} + 7n^{\mathrm{viii}} = 0 \quad . . . . . (3).$$

"The re-examination of the last of these has recently led to the discovery of two others, viz:—

$$68n^{\mathrm{vi}} - 325n^{\mathrm{vii}} + 257n^{\mathrm{viii}} = 0 \quad . . . . . (4).$$
$$257n^{\mathrm{v}} - 844n^{\mathrm{vi}} + 587n^{\mathrm{vii}} = 0 \quad . . . . . (5)."$$

. . . . . "The fifth, however, is not an independent equation, but is derived from the third and fourth. . . . . It is obvious, moreover, from the same equations, that *no three of the four outer planets can ever be in conjunction at the same time.*"

The more thorough revision indicated by Prof. Peirce would be requisite before *all* these relations could be definitely settled; but they furnish additional occasion both in the planetary system and in that of Saturn for the explanation which M. Laplace himself has given, in Note VII to the *Système du Monde*, of the special relation apparent in the first of the instances here quoted, viz., that of Jupiter's satellites.

That illustrious astronomer indicates that "in order to produce the equation with regard to those satellites, already quoted, it would be sufficient that, at first,

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[1] *Outlines of Astronomy* (11th edition), (550).
[2] At p. 208 of the same volume.

there should have been a very close approximation to the conformity in question, and then the mutual attraction of the satellites would rigorously establish such a conformity;" and hence, moreover, "make the mean longitude of the first satellite *minus* three times that of the second, *plus* twice that of the third, always equal to a semi-circumference."

At the same time, as he says, this would originate a periodical inequality dependant on the small quantity by which the mean motions "primitively deviated from the relation which we have announced. Notwithstanding all the care which Delambre took to make out this inequality by observation, he could not discover it; which proves its extreme minuteness, and consequently indicates with very great probability a cause which made it disappear."

M. Laplace then proceeds to show that, on his own hypothesis, the satellites of Jupiter, immediately after their formation, did not move in a perfect vacuum; but that the less condensable molecules of the primitive atmospheres of the sun and of the planet *furnished a resisting medium*,[1] the effect of which would be different on every one of the satellites in question, and when their motions attained the conditions requisite to the establishment of the conformity of motions, the same resistance diminished the inequality to which this relation gave rise, and finally rendered it insensible.

All this may well be extended to the case of *the conformity of periodic-times in Saturn's system*, as well as *those of the periodic-times of the outer planets* already specified.

M. Laplace illustrates the process in question by the retarded motion of a pendulum in a resisting medium; entire revolutions being reduced to oscillations diminished continually by the resistance of the medium, and in the end annihilated; the pendulum coming to rest, and ever after remaining so.

The original passage in which this illustration occurs, is the closing one of the *Système du Monde;* and is as follows:—

"On ne peut mieux comparer ces effets, qu'au mouvement d'un pendule animé d'une grande vitesse, dans un milieu très peu résistant. Il décrira d'abord un grand nombre de circonférences; mais à la longue, son mouvement de circulation toujours décroissant se changera dans un mouvement d'oscillation, qui diminuant lui-même de plus en plus, par la résistance du milieu, finera par s'anéantir; alors le pendule arrivé à l'état du repos, y restera sans cesse."

The changes indicated in the quotation in the next article, contemplate a veritable oscillation, in some measure like this.

Special Characteristics of the Moon, and other Satellites.

(68) M. Laplace, commenting on his own hypothesis, in the connexion already referred to, (67), thus expresses himself: "One of the most singular phenomena of the solar system is the rigorous equality observed between the angular motions of rotation and revolution of every satellite. We may wager infinity to one that

[1] The italics are our own.

this is not due to chance. The theory of gravitation causes the infinity of this unlikelihood to disappear, by showing us that, for the existence of the phenomenon, it would be sufficient that the motions should have been very little different at their origin.[1] Then the attraction of the planet established between them a perfect equality; but at the same time gave rise to a periodic oscillation of the axis of the satellite directed toward the planet, the extent of it dependant on the primitive difference of the two motions. The observations of Mayer on the libration of the moon and those which MM. Bouvard and Nicollet made with reference to this matter, at my request, have failed to make known this oscillation. The difference on which it depends must, therefore, have been very small; which indicates, with extreme probability, a special cause which first kept this difference within the very narrow limits within which the attraction of the planet could establish an equality between the mean motions of rotation and revolution, and which afterwards destroyed the oscillation which this equality had originated. Both these effects result from our hypothesis. For it will be understood that the moon in the state of vapors, formed, because of the powerful attraction of the earth, an elongated spheroid the major axis of which must be incessantly directed towards that planet, from the facility with which vapors yield to the smallest force which animates them. The terrestrial attraction continuing to act in the same manner when the moon was in a fluid state, at length, in approximating incessantly the two motions of this satellite, caused them to fall within limits such that their rigorous equality began to be established. Afterwards this attraction must, little by little, have annihilated the oscillation which this equality produced in the axis of the spheroid directed towards the earth."

"It is thus that the fluids which covered this planet[2] have destroyed, by their friction and their resistance, the primitive oscillations of its axis of rotation, which now is subjected but to the nutation resulting from the actions of the sun and the moon. It will be readily seen that the equality of motions of rotation and revolution would present an obstacle to the formation of rings and of secondary satellites from the atmospheres of those bodies. Accordingly, observation has thus far indicated none such."

(69) It is claimed that the other satellites of the planetary system resemble the moon in the coincidence of their times of rotation and revolution; and thus presenting always nearly the same side of any satellite toward its primary. This is inferred from special vicissitudes of the light of the satellites *recurring* when they have again arrived *at the same positions* in their orbits around their respective primaries.

Nor is that all. Among the remarkable phenomena presented by satellites is that of their seeming loss of light; all Jupiter's satellites, having, at times, been seen to transit the disk of the planet, appearing, in whole or in part, as *dark* instead of *bright* spots; and that sometimes after having *first* appeared *bright* and then *dusky*.

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[1] In this connexion, *see*, again, *Note* on p. 22.

[2] The Earth.

This—as has elsewhere been indicated by the author of this paper—would seem to be due to the absorption, and, possibly also, to the interference of light on a scale such as Astronomy alone exhibits; of the light, viz., reflected from Jupiter and meeting that of the satellite.

(*a*) Aside from all that, however, the phenomenon, or rather phenomena, in question would seem to be consistent with the conclusion of a coincidence in the times of rotation and revolution; for the appearance of the satellite, in the course of its transit, as a black spot has, within moderate intervals of succession, recurred when the satellite had returned to a like position in its orbit around its primary.[1]

(*b*) Admitting the absorption already indicated; then, instructed by the revelations of the spectroscope, we may regard it as probable that the satellite must be *colder* than its primary.[2]

(*c*) This last would happen—indeed we would have a reason for it—if the satellite, like the moon, had *little or no atmosphere.*

(*d*) All these analogies would be quite consistent with the hypothesis that all these satellites (including the moon) had been similarly condensed from the nebulous state, and then subjected to the stringent conditions which prevail in satellite systems. The loss of atmosphere is one of the supposable consequences of those stringent conditions; as indeed M. Laplace has intimated, when after stating the distance at which the attractive force of the earth is in equilibrium with that of the moon, he adds: "If at this distance, the primitive atmosphere of the moon had not been deprived of all elasticity, it would be carried to the earth, which could thus draw it to itself, (*aspirer*). This is, perhaps, the reason why the moon's atmosphere is so nearly insensible."[3]

Of the Zodiacal Light.

(70) As to the region of the zodiacal light; M. Laplace, in speaking of the atmosphere of the sun, says: "The atmosphere at the equator cannot extend beyond the point where the centrifugal force exactly balances gravitation; for it is manifest that beyond that limit the fluid must itself be dissipated. As respects the sun, this point is at the distance from his centre of the radius of the orbit of a planet which would complete its revolution in a time equal to that of the rotation of the sun. The atmosphere of the sun, therefore, does not extend even to the

[1] But the conclusion is not a necessary one. M. Secchi makes the time of rotation shorter than that.

[2] Some recent observations of Jupiter seem to indicate that the planet itself is highly heated—possibly even to the extent of being locally self-luminous. The color of the belts and its variations together seem consistent with all this. [Witness the exquisitely beautiful chromo-lithographs accompanying the Earl of Rosse's paper in No. 5. of vol. XXXIV, of the *Proceedings of the Royal Astronomical Society;* and Mr. John Browning's very beautiful representations of similar phenomena in No. 9 of the same volume. Also M. Tacchini's very remarkable diagram of Jupiter's appearance; with his explanations (*Comptes Rendus*, tome LXXVI, p. 423).]

[3] Conclusion of Chap. X, of Book IV, of the *Système du Monde.* For a discussion and an explanation of the various phenomena here in question, see two communications, by the author of this paper, to the *Astronomische Nachrichten*, Nos. 1986 and 2012.

orbit of Mercury, and, consequently, it does not produce the zodiacal light, which seems to extend even beyond the earth's orbit. Moreover this atmosphere, whose polar axis must be at least two-thirds of that of the equator, is very far from having the lenticular form which observations give to the zodiacal light."[1]

(71) Next as to the origin and the constitution of the material which gives us the zodiacal light, we have: "If, among the zones abandoned by the atmosphere of the sun, there should be molecules too volatile either to combine themselves, or to unite with the planets, they ought, while continuing to circulate about the sun, to present all the phenomena of the zodiacal light without opposing a sensible resistance to the diverse bodies of the planetary system, either because of the extreme rarity of those volatile molecules, or because their motion is very nearly the same with that of the planets which they encounter."[2]

It will be observed that the first of the two quotations, here made, intimates it as probable that the material from which the Zodiacal Light proceeds, itself extends beyond the earth's orbit. This is, in fact, intimated by the existence of what in German accounts of observations of the Zodiacal Light has been designated as the *gegenschein;* which is seen in the part of the heavens opposite to the sun; the existence of which phenomenon is established by numerous observations, such especially as are detailed in various numbers of the *Astronomische Nachrichten.*

(72) Both eastern and western appearances occurring simultaneously are reported by the late Rev. George Jones, A.M., chaplain in the U. S. Navy; these phenomena being, among numerous others, the description of which, and other things connected with them, itself occupies the whole of vol. iii. of the *Report of the U. S. Japan Expedition;* and the extent of the light to *both sides of the heavens* is confirmed by the observations of Col. Charles G. Forshey, U. S. A., made while he was stationed in an elevated and dry region of Texas; where, as stated by Col. Forshey to the author of this paper, that phase of the phenomenon was a common occurrence; though the appearance of the Zodiacal Light in lower Louisiana, as described by him, was very different.[3]

(73) All this makes it more difficult to admit that the material in question can be maintained in position, with the sun for its centre of reference; the conservative

[1] *Système du Monde,* Book IV, Chap. X. [2] *Système du Monde,* Note VII.

[3] In Col. Forshey's manuscript notes, which he has since confided to me, the Zodiacal Light is described as being "very distinct across the heavens," Nov. 10, 1858, at 10 o'clock P.M. As delineated on star charts, the outlines on this occasion, as on many others, approach to a hyperbolic form, the central line of the luminous band being in the position of an asymptote to the two edges; or—if the comparison may be allowed—the appearance often was that of an enormous trumpet, the lower end widening rapidly and extensively; and on the occasion here referred to, *two* such appearances are delineated, as having been observed; the broad ends spreading out to the horizon, on opposite sides, and the narrow portions united midway.

On the 9th of May, 1860, the phenomenon is described as being "faintly visible across the canopy;" though the whole display is characterized as being "rather faint;" while the "evening" is noted as being "splendidly clear."

Also Nov. 13, 1859—"Not a very bright display. Still column very distinct all the way across the sky."

And, in a "Note" under the date of March 31, 1858, Col. Forshey expressly says: "I now begin to think that well-trained eyes can see it all the way round, at all times that are clear and moonless."

influence of the great planets being not supposable within the extended limits of the solar system; though the satellites of Saturn, [*Note* [3] to 7 of (43)], are efficient in that way, maintaining the position of the rings, under the more stringent conditions of a closer arrangement.

Added to this, is the consideration of the enormous extent which would seem to be required on both sides of the ecliptic, to account for the great breadth of the *base* of the zodiacal illumination, even after the disappearance of twilight in the evening, or before daylight in the morning; all which seems to be true of *the more dense*, and, if surrounding the sun, also *the more distant portion* of the material in question, which ought, unless uncommonly extensive, to be seen *under a smaller angle* than the other portions of the same; a difficulty to which the hypothesis recently advanced by Mr. Richard A. Proctor, F.R.A.S., viz. that the Zodiacal Light is due to a closely arranged group of meteors, would seem to be especially liable; and all the more so, if "assuming" (as he himself says we are bound to do) "a considerable degree of flatness in the actual figure of the zodiacal disk, and more especially of its more distant portions."[1]

And just *that* difficulty still remains if we were even to *admit* Prof. Arthur W. Wright's conclusion from his recent experiments on the *polarization* of the Zodiacal Light, as far as this—that "the light is reflected from matter in *a solid state*;" since, he adds, in explanation of the same that this solid matter is that of "innumerable small bodies revolving about the sun in orbits of which more lie in the neighborhood of the plane of the ecliptic than near any other plane passing through the sun."[2]

Now this portion of the hypothesis of Prof. Wright, Mr. Proctor, and, it may be, others—whatever may be the special composition of the material in question—would seem to require that the apparent form of the Zodiacal Light should be somewhat like that of the head of a comet, with the *expansion* beyond it extending *upward* from the sun; whereas the actual appearance and position are both the reverse of that; the *broad* base near the horizon, and the *narrow* and curved termination at *the upper* end.

And then, moreover, it would seem, on the part of the hypothesis here considered, that, in any event, there must be a conspicuous central beam or core of the Zodiacal Light; which we do not find.

And, lastly, what shall be said of the *planetary perturbations*, which, it would seem, ought to be superinduced by such a closely arranged group of meteors; especially if the "light" be indeed "reflected from matter in a *solid* state?"

Other objections to hypotheses which would make the material to which we owe the Zodiacal Light to be an appendage of a lenticular or other form, referable to the sun as its centre, are very exhaustively considered by Chaplain Jones in the volume already referred to. The hypothesis that the Zodiacal Light is due to

[1] In a long and carefully considered *Note on the Zodiacal Light* in the *Monthly Notices of the Royal Astronomical Society*, vol. xxxi, No. 1 (Nov. 11, 1870).

[2] *American Journal of Science and Arts, Third Series*, vol. vii. p. 457 (No. 41—May, 1874). Will, after all, our terrestrial experience as to the conditions of *polarization*, justify us in making *it* a *criterion* of the *state* of anything so peculiar as the matter in question?

reflection from the earth's atmosphere is also discussed and rejected by him. Upon this, however, it will not be necessary here to comment; as it, most probably, is no longer insisted upon by any one.

(74) It remains, then, to consider *with what modifications* we may admit Mr. Jones's hypothesis; that the nebulous material which gives the Zodiacal Light is a terrestrial appendage; and also what is the conservative force, which may insure its preservation of form, and its maintenance in its revolution around the earth, even in close proximity to the moon.

Antecedent to all that, however, will be found to be the questions of density and of mode of illumination, as well as, in its proper connexion, the question of parallax.

The density of the material in question seems indeed to be that intimated in the description of M. Laplace already quoted, (71); viz. that which pertains to the state of molecules "too volatile either to combine themselves, or to unite with the planets." And this is confirmed by the spectrum-analysis; the result of which has led to no other reliable conclusion than that of the extreme rarity of this same material.[1]

This same rarity of the material in question is withal indicated by its transparency.

Of this Rev. George Jones says, under date of Dec. 30, 1854 (in lat. 10°46′ N., long. 89°31′ W. of Greenwich): "I also, this morning, gave attention to the stars as seen through the Zodiacal Light, and found, even to $4^h\,30^m$, when the effulgent light below the zigzag lines (in the chart) is very strong, that with the naked eye I could readily make out stars of the 6th magnitude within the effulgent light; . . . . . also a line of four stars below 19 Libræ, and ranging with β Libræ; . . . . . the two northernmost of these last are of the 7th magnitude, yet I think the naked eye detected them, even within this effulgent light; but the last are near its upper edge. All this shows the great transparency of the substance giving the Zodiacal Light."[2]

(75) The consideration of these phenomena leads to the conclusion, That this light proceeds from particles which, as respects size, are, at most, all but *molecular*, and if discrete, and, possibly, "solid," yet *excessively* small solids. It then must also largely be *transmitted* light; and so the illuminated material *appear brighter in the special direction in which the light is transmitted.* Chaplain Jones illustrates this in part, when he says that "it seems to be quite conclusive, on an inspection of these charts, that *we never at any one time see the whole actual extent of the Zodiacal Light.* This subject can, perhaps, be elucidated by noticing a common event—a cloud silvered at one edge by the rays of the declining sun. The sun may be shining on the bordering, quite around that cloud; and, if so, it is sending off from every portion of the border, an equally brilliant silvery light. But our eye is in a position to

[1] Such is in effect the statement of Prof. Charles A. Young (as the result of his experience and that of others), made in a personal communication with the author of this paper.

[2] Report of Japan Expedition, vol. iii, No. 271, at p. 542.

catch this reflection from only one portion of it; and the rest is dull to our vision. If we could with great rapidity change our positions, other portions of the silvered edge would show themselves according to our changes of place. So also, when a rainbow is presented to our eye; the myriads of drops of falling water in the whole rain-shower are sending off from each drop reflections of light in all directions, and the universal atmosphere about us is full of these brilliant variously-colored rays; but only that portion, which, to us, forms the rainbow arch, can reach our eye; and all the rest is lost to our sight."

"So it is also with the Zodiacal Light; and the proof that we never see the whole of its extent at once, is manifest in the following facts:—

"1. When I was in a position *north* of the ecliptic, the main body of the Zodiacal Light was on the *northern* side of that line.

"2. When I was *south* of the ecliptic, the main body of the Zodiacal Light was on its *southern* side.

"3. When my position was *near* or *on* the ecliptic, this Light was equally divided by the ecliptic, or nearly so.

"4. When, by the earth's rotation on its axis, I was, during the night, carried rapidly to or from the ecliptic, the change of the apex, and of the direction of the boundary lines, was equally great, and corresponded to my change of place.

"5. That, as the ecliptic changed its position as respects the horizon, the entire shape of the Zodiacal Light became changed, which would result from new portions of the nebulous matter coming into position for giving us visible reflection; while portions lately visible were no longer giving us such reflection."[1]

(76) The phenomena here commented upon all serve to confirm the assertion, (75), that the zodiacal illumination must largely be transmitted light; and so the illuminated material *appear brighter in the special direction in which the light is transmitted;* as the sun illuminates the partially transparent vapor in our atmosphere through rifts in the clouds, and thus produces the appearance familiarly described as "the sun drawing water."[2]

(77) The light being transmitted, other phenomena would also be in place, among which are *absorption*—possibly *interference*—and also *fluorescence;* new waves being originated in this case, as well as, perhaps, in that of the comets; the spectrum-analysis of whose light seems to show, among other phenomena, characteristics of self-luminous material.

(78) To this it may now be added, that the nebulous ring of Chaplain Jones, may well be regarded as having, indeed, not the *lenticular form* attributed to the

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[1] "The first four of these results were not always uniform; but the exceptions were few, and were probably occasioned by the nebulous ring's not lying exactly in the plane of the ecliptic." From the *Introduction* to Chaplain Jones's Report, pp. XVI and XVII.

[2] Mr. Proctor also seems inclined to admit the possibility of a more intense illumination in special directions; though not decided as to its cause, when he says at the close of his Note on the Zodiacal Light, referred to in (73): "If some solar action, for example, rouses luminosity in certain definite directions—as, for instance, near the plane of the Sun's equator—in some such way as light is caused to appear along radial lines through and beyond the heads of comets, our power of theorizing from such considerations as have been dealt with in this paper would be limited."

material giving the zodiacal light by older hypotheses (which he does not claim); nor yet that of a ring like those of Saturn; nor yet a ring of greater thickness, partially luminous indeed in appearance, as Mr. Jones would have it; but we must have for it the form of what may rather be termed *a girdle*, of *no great thickness*, it may be—it is too translucent for that—but yet of very considerable width, such as will provide for the broad base of the Zodiacal Light, and the extended elliptical spot which exhibits the "*gegenschein*"[1] opposite to the sun; and which latter would seem to be almost wholly due to reflection. There may also be some reason to suppose that the curvature of the girdle, on the one side at least (that on which the "gegenschein" appears) is such as would be due to a spheroidal shell such as has been described in *Article* (37) of this paper. Such a girdle, withal, could not always—perhaps ever—have all its breadth enveloped in *the earth's shadow*.

How the Girdle is maintained.

(79) The question at once becomes a pertinent one, How can such a girdle escape destruction by the continued perturbation of the moon, acting in close proximity?

The answer to this question may be found, if the girdle be so situated that ITS TIME OF REVOLUTION AROUND THE EARTH SHALL BE EQUAL TO, AND IN THE SAME DIRECTION WITH, THAT OF THE MOON. The conditions requisite to fulfil this will first be considered, and then the phenomena that seem to be accordant with the actual maintenance of such conditions.

(80) If the *earth's* attraction *alone* were concerned, the form of the revolving girdle must, it would seem, be that of a spheroidal shell; such as that indicated in (37). The attraction of the moon will distort this, yet so that the shape shall also be consistent with the stringent condition as to the periodic time.

Fig. 13.

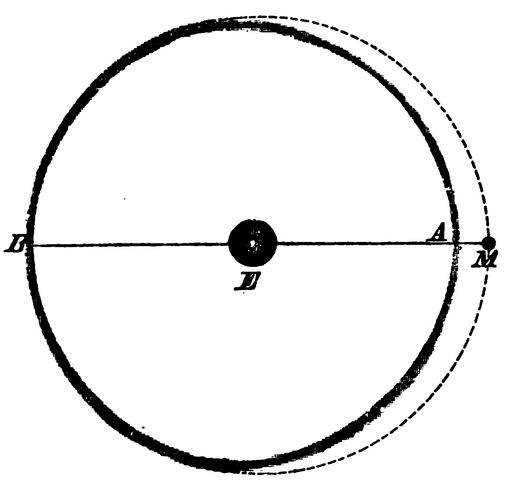
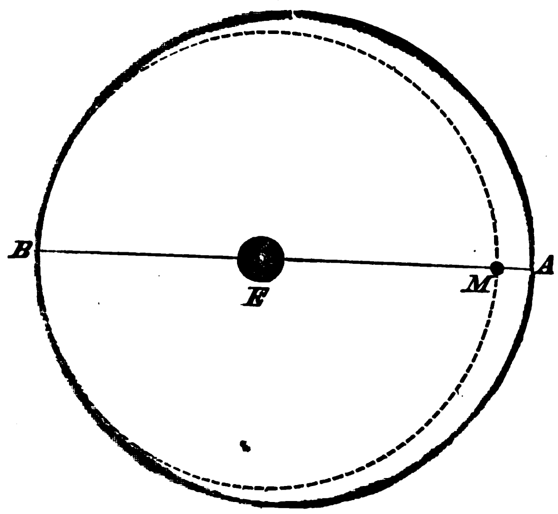


Fig. 14.



[1] *Counter-gleam*, we might perhaps term it; though that scarcely seems so apt as the German word for the same thing, here quoted.

The middle line of the girdle will, notwithstanding, form an oval, which, at any time, in its arrangement around the earth, will not anywhere be found at a distance differing much from that of the moon at that time; except in those portions comparatively near to the moon.

That part of the oval nearest the moon may pass between the moon and the earth, as in *Fig.* 13; or else outside of the moon, as in *Fig.* 14; in both of which E marks the position of the earth, and M that of the moon.

In the determination of the dimensions in either case, it will be convenient to ascertain the periodic time of a particle, or of an inappreciable mass, revolving around the earth at the mean distance of the moon; which we may obtain by the aid of the following formula, in which (T) will be put for the periodic time; M and m representing the masses in question, and r the radius-vector; and we have

$$(T) = \frac{2\pi r^{\frac{3}{2}}}{\sqrt{M+m}} \quad \ldots\ldots \ (1).$$[1]

Then, when m is insensible,

$$(T') = \frac{2\pi r^{\frac{3}{2}}}{\sqrt{M}} \quad \ldots\ldots \ (2);$$

and, when r is the same for both, from these we also have,

$$\frac{(T')}{(T)} = \frac{\sqrt{M+m}}{\sqrt{M}} \quad \ldots\ldots \ (3); \text{ or}$$

$$(T') = \frac{\sqrt{M+m}}{\sqrt{M}}(T) \quad \ldots\ldots \ (4);$$

which, otherwise expressed, is

$$(T') = \sqrt{\frac{M+m}{M}} \cdot (T) \quad \ldots\ldots \ (5).$$

Then, 1*st*—Making special application of either *Eq.* (4), or *Eq.* (5) to the example in which M and m, respectively, represent the masses of the earth and the moon, and (T) the moon's periodic time, we shall have the periodic time of a particle, or of an insensible mass, revolving around the earth at the distance of the moon.

2*d.* Ascertain the periodic time (t) of the same insensible mass, revolving about the earth, at the assumed distance EA, by the application of *Kepler's* 3*d Law.*

3*d.* The attractive forces of the moon and the earth, respectively, acting at A may be separately computed in accordance with the law of gravitation $\left(\frac{M}{d^2}\right)$, and then taking the *difference of* the two forces, when the state of things is that represented in *Fig.* 13; and expressing this difference in terms of the earth's force F, viz. as $\frac{p}{q}F$; then (with (t), the periodic time around the earth of an insensible mass revolving at distance EA, *already computed*), we shall have

[1] Encyclopædia Metropolitana—*Physical Astronomy,* Section V.

$$\frac{(t')^2, \text{ for } \frac{p}{q}F}{(t)^2, \text{ for } F} = \frac{F \text{ itself}}{\frac{p}{q}F}; \text{ whence}$$

$$(t')^2 = \frac{F}{\frac{p}{q}F} \cdot (t)^2; \text{ and}$$

$$(t') = \left\{ \frac{F}{\frac{p}{q}F} \cdot (t)^2 \right\}^{\frac{1}{2}}$$

Then if (t'), thus computed, be found to be equal to the moon's own periodic time, the point A will have been accurately ascertained; the particle, or the insensible mass (in the line EM), completing its revolution at the distance EA, in the same time with the actual revolution of the moon around the common centre of gravity of the moon and the earth.

But if (t') differ at all from *that*, the difference may be exhausted by the continued application of the method of trial and error.

When A is situated beyond the moon (in accordance with the representation in *Fig.* 14) the *sum* of the attractive forces of the two bodies must be made to enter into the equation to determine the value of (t'), instead of the difference of those same forces. So also, for the distance from E to B, on the opposite side of the earth.

(81) Now the division or the extension of EM (as the case may be) so as to give the distance EA, this depends upon the forces in question, and, ultimately, on the ratio of the masses, and not upon the absolute length of EM. Hence EA and EB will each have a constant ratio to EM; whether the moon be in apogee, or in perigee, or at the mean or any other distance. The same is true of the distance of the moon from the common centre of gravity of the moon and the earth, *i. e.* of the *radius-vector of the moon's orbit;* and for the same reason.

Now,—(*a.*) Every other of the quantities in question having, after this manner, a constant ratio to EM; it will follow that, under all their variations of value, the value of any one of the quantities will preserve a constant ratio to the coexistent value of any other; and therefore, specifically, to the coexistent value of the *moon's radius-vector;* or the square of the one, a constant ratio to the square of the other.

(*b.*) Next, as M, E, A, and B, under the conditions in question, are preserved in the same straight line; it follows from the doctrine of parallels, that *the angular change of direction* of M revolving about the common centre of gravity of M and E, or that of A and B revolving about E, will be *the same* with reference to any fixed direction in space, such as that of EM (at any instant), or with reference to its parallel; or the same will be true with respect to the first tendency to such change, *i. e.* its *differential.*

(*c.*) Hence also, especially, the angular change of direction which would take place, were such a tendency preserved during the next *unit of time*, *i. e. the co-*

existing angular velocity of *M*, *A*, *B*, (in their revolution of every one of them around its centre of reference) would, in every instance, have the same value.

(*d.*) But this *same* angular velocity in the moon's orbit varies inversely as the square of the *radius-vector*, and the coexisting values of the squares of *EA* and *EB*, respectively, having (as already shown) constant ratios to *that;* their ratios may be substituted for the ratios of the respective coexisting values of the squares of the radii-vectores themselves; and the inversion of the one for the inversion of the other.

(*e.*) By substitution, then, the respective squares of *EA* and *EB* are inversely as the coexisting angular velocities in the moon's orbit.

(*f.*) But the same angular velocity being (as also shown) common to all the three masses in question; every one of those masses will also have its angular velocity inversely as the square of its own radius-vector; and that will imply the principle of *the conservation of areas;* and thus maintain not only for the moon, but also for the other masses, in the consentaneous revolution of all, *a dynamical equilibrium.*

(*g.*) Then withal the constancy of the ratios already specified, will secure, under the coexisting similar change of angle, the same ratios among the radii-vectores of all the three trajectories here in question; and just all that implies that the *same polar equation* will apply to all the three.

(*h.*) Hence the trajectories of *A* and *B* are both *ellipses;* as well as (perturbations apart) is the orbit of the moon; even more than this, under those stringent conditions (common to all); viz. the trajectories are all *similar* ellipses.

(82) The positions of the points *A* and *B*, on the supposition that the girdle on the one side, is between the earth and the moon, as in *Fig.* 13, is exhibited in the following table; the distances represented being in terms of the earth's equatorial radius.

	IN PERIGEE.	AT MEAN DISTANCE.	IN APOGEE.
MOON'S DISTANCE......................	56.964	60.273	63.583½
(*EA*) Internal Distance of Girdle........	48.309	51.116	53.922½
(*EB*) External Distance of Girdle........	56.790	60.090	63.389

On the supposition that the girdle encompasses the moon, as in *Fig.* 14, we have:—

	IN PERIGEE.	AT MEAN DISTANCE.	IN APOGEE.
MOON'S DISTANCE......................	56.964	60.273	63.583½
(*EA*) External Distance of Girdle........	66.426	70.285	74.144½

(83) As *A*, *B*, and the moon thus describe similar ellipses with their radii-vectores coincident in the same straight line; it is manifest that the portions of the girdle in the immediate neighborhood of *A* and *B* will *expand* (the material

being readily adjustable) as the moon passes from perigee to apogee; and they will contract as the moon passes from apogee to perigee; the cohesive power and the gravitation of outer to inner portions being, in any event, insensible; and so each particle or molecule moving in its independent, or nearly independent, ellipse very much as Sir J. Herschel has intimated that the molecules of comets might move.[1]

Then, too, a *permanent* tide must influence and control the form of the girdle; this tide (with the arrangement as in *Fig.* 14) being in some sense *supra*-lunar, instead of *sub*-lunar, in the region of the crest of the girdle extending beyond the moon.

By such a tidal action an *accumulation of material* will be determined toward the *two extremities* of that axis of the girdle, which at any time passes through the two centres—that of the earth and that of the moon—and which is extended to the girdle on both sides [*i.e.* toward A and B in either of the cases represented, the one in *Fig.* 13, and the other in *Fig.* 14].

And the portions of the adjustable material here specified having themselves been once so adjusted (radii-vectores and all) as to be held, or very nearly held, in a dynamical equilibrium, such as is specified in (81); the compulsory power of the forces acting on such material, under such stringent circumstances, might well be supposed to bring about the form required to secure a dynamical equilibrium of the girdle; though the oscillations, in various directions, antecedent to that, would present a problem of no ordinary difficulty.

However all that may be—the dynamical equilibrium of all parts of the girdle being once established, the state of things afterward would be eminently conservative of the same; such being especially the case with respect to *the various* actions, which, under other conditions, might be eminently destructive.

(84) If the girdle (as at A in *Fig.* 13) were between the moon and the earth, *its* curvature would be *diminished* in the direction perpendicular to the moon's orbit, by the moon's own action; though the curvature would be *increased* by the action of the moon, on the opposite side; as was, indeed, intimated, though not at all explained, in (78). But if the girdle (as at A in *Fig.* 14) were outside of the moon, the curvature (perpendicular to the moon's orbit) would be greater still.

(85) The second thing proposed in this connexion, was to consider the phenomena which seem to be accordant with the state of things thus far represented as being merely supposable. With respect to these phenomena, it may be observed, that the hypothesis of the girdle having the same periodic time with the moon suggested itself as a necessity, to insure the preservation of the girdle itself; and, in the brief interval which has since elapsed, the variations of the Zodiacal Light have, to some extent, been carefully noted, and *then* referred for explanation to the hypothesis.

And here the phenomena seem to be more consistent with the arrangement of the girdle as represented in *Fig.* 14; the point A being situated *beyond* the moon.

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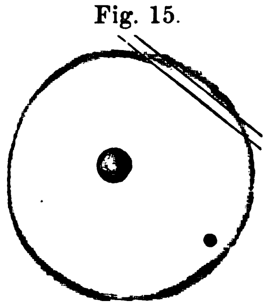
[1] Cabinet Cyclopædia—*Astronomy* (488).—With this Prof. Wright's conclusions, (73), with respect to the *constitution* of the material in question would not be inconsistent. See, again, *Article* (73).

With that in view, the special appearances of the Zodiacal Light may be arranged as follows:—

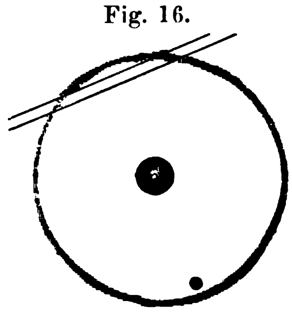
Case 1*st*. The Zodiacal Light appears narrow and towering high just about the time of the new moon; *as though the sun's light were indeed transmitted, at that time, through the least curved, and, probably, somewhat rarer sides of the oval-shaped girdle; and that through a great part of the length of the oval.* (*Fig.* 15.)

Fig. 15.



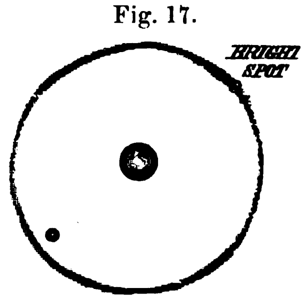
Case 2*d*. After the new moon, when the moon is approaching her first quarter; when the moon has set, and the twilight has disappeared, the Zodiacal Light does not extend so high as in the preceding case, and its termination is broader, and not so sharply curved, and the intensity of the light, withal, is not especially conspicuous (as in *Fig.* 16, for Zodiacal Light of the morning), *as though the sun's light indeed. in all its transmission, passed through the rather less dense portion of the girdle; and passed out of it in a direction more across the girdle and not so nearly at a tangent to it (in its exit under these circumstances), as in the preceding case.*

Fig. 16.



Case 3*d*. After the full moon, and when the moon is approaching her last quarter; then, before the rising of the moon, and after the end of twilight, a luminous spot of considerable size, and, in appearance, like the brighter portion of an aurora borealis, occupies the place in the Zodiacal Light which is quite accurately opposite to the moon's place; and night after night, as the moon advances, this luminous spot rises among the stars, so as still to keep opposite to the moon; *as though the somewhat more dense portion at the farther end of the oval (as respects the moon) were thus more conspicuous than the other portions then in view; and then the upper extremity of the Zodiacal Light is broader and not so sharply pointed as in Case* 1*st; as though for the reason assigned in Case* 2*d.* (*Fig.* 17.)

Fig. 17.



Case 4*th*. After the last quarter and before the new moon, the Zodiacal Light of the evening is again faint, as it was before the first quarter; *as though the illumination were wholly of that part of the girdle beyond the region near the longer axis.* (*Fig.* 18.)

Case 5*th*. When the moon is nearly in quadrature, it would seem that the Zodiacal Light must appear *short and bright*, if apparent at all after the twilight of the evening, or before the twilight of the morning. For the sun's light would be transmitted by a short course through the most curved portion, near to one end of the longer axis of the oval. (*Fig.* 19.)

(86) Increase of brightness might be looked for, with the moon in perigee; and of extent, with the moon in apogee. Traces of something like one and the other have been apparent.

(87) After an examination of Chaplain Jones' very numerous charts, a selection was made of those which seemed to exhibit instances in which the light was most

Fig. 18.

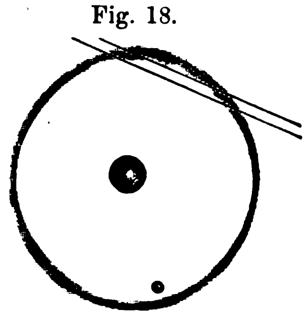
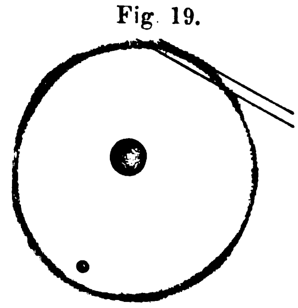


Fig. 19.



extensive, or most conspicuous, and others in which, in one or both respects, the light seemed to be deficient (the character of the light, and not the position of the moon, furnishing the guide in the selection); and *then* the age of the moon, and her position in her orbit were ascertained, for a comparison of the phenomena with theory.

The following instances were then classified with reference to our hypothesis now under discussion. The *Nos.* are those of Mr. Jones' charts:—

Examples under Case 1st.

No. 219.—Morning of Sept. 21, 1854; 1 day before new moon.
No. 220.—Evening of Sept. 23, 1854; 1 day after new moon.
No. 232.—Morning of Oct. 20, 1854; 1 day before new moon.
No. 233.—Morning of Oct. 21, 1854; the day of new moon.
No. 243.—Morning of Nov. 21, 1854; 1 day after new moon.
No. 259.—Morning of Dec. 19, 1854; the day of new moon.

(A very marked instance; and not only was the day that of *new moon*, but the moon was also *in perigee.*)

Mr. Jones, without any reference to the moon's age, or to her distance from the earth, says of the zodiacal light, "At 2h. the eastern zodiacal light was bright, at 3h. 30m. quite so. At 5h. it was as brilliant as I have ever seen it, and was especially so within the zigzag" (waving lines toward the lower part of the diagram), "where the light had more of a cone shape than I ever saw it have before.[1] . . . . Sun rose at 6h. 57m."

Approximation to Case 1st.

No. 49. Morning of Sept. 2d, 1853; 1 day before new moon.

Examples under Case 2d.

No. 31. Evening of July 9th, 1853; 3 days after new moon.
No. 114. Morning of Feb. 1st, 1854; 3½ days before first quarter.

[1] The description here is such as might, in anticipation, have been dictated by the hypothesis under discussion.

Case 2d, or Case 4th.

No. 161.—Evening of May 29th, 1854; 3 days after new moon.
No. 237.—Morning of October 30th, 1854; 2 days after first quarter.

Examples under Case 3d.

No. 212.—Evening of Sept. 12th, 1854; 1¼ day before last quarter.
No. 213.—Evening of Sept. 13th, 1854; ½ day before last quarter.

Examples under Case 4th.

No. 18.—Evening of June 29th, 1853; 1½ day after last quarter.
No. 60.—Morning of Sept. 30th, 1853; 2½ days before new moon.
No. 215.—Evening of Sept. 16th, 1854; 2 days after last quarter.

Examples under Case 5th.

No. 67.—Morning of Oct. 8th, 1853; 1 day before first quarter.
No. 214.—Evening of Sept. 14th, 1854; day of last quarter.
No. 239.—Evening of Nov. 11th, 1854; 1 day before last quarter.
No. 241.—Evening of Nov. 13th, 1854; 1 day after last quarter.[1]

(88) Mr. Jones also gives examples of "Moon Zodiacal Light."

(89) Baron Humboldt, commenting on Rev. Mr. George Jones's observations, quotes from his own ship-journal on his voyage from Callão to Acapulco, and speaks of the brilliancy of the Zodiacal Light as exceeding anything which he had previously witnessed. The time when this was observed was from the 17th to the 19th of March, 1803. Indeed the intensity of the light increased for five or six nights after the 14th. Height 39° 5'.[2]

As the moon was new on the 23d, this bright light must have begun before the last quarter; and will present a probable instance of *Case 3d*, passing into and beyond Case 5th.

But, strangely enough, Baron Humboldt finds occasion to add: "We did not see the Zodiacal Light the 20th and 21st of March, *although the nights were of greatest beauty*."

Now something—perhaps not a little—of that may have been due to differences in the state of moisture of the atmosphere, such as those, (72), of which Col. Forshey has informed us. But the time being withal from two to three days before the new moon, the sun's light would, on the hypothesis here in question, be transmitted through the curved portion of the girdle a little in advance of the longer axis.

The length of the transmitted portion would not be great, and the upper end would set almost as soon as the twilight ended.

(90) In the account of Prof. C. Piazzi Smyth, Astronomer Royal at Edinburgh, of his expedition to Teneriffe, under date of Aug. 19th, 1856, speaking of the Zodi-

[1] See *Astronomische Nachrichten*, No. 989.
[2] The dates with reference to the phases of the moon are but close approximations; yet such as are quite sufficient.

acal Light, he says: "So bright was it toward the base that it produced a weak reflected glow to the west, and we could occasionally fancy a tail of the faintest conceivable light extending nearly to the zenith." (Length of the bright light was 63°.) "Nevertheless there was no doubt of the lenticular form of the chief mass of light, and the place of its apex as measured, was always consistent enough."[1]

This was almost three days after the full moon, and seems to present an example of *Case* 3d. Under the date of Sept. 8th, Prof. Smyth says of the Zodiacal Light—"bright at base, glowing toward the lower part of the axis."[2]

This was one day after the first quarter of the moon; and we here would seem to have an example of *Case 5th.*

(90 *bis*) The observations of Col. Charles G. Forshey, already alluded to in (72), were made while Col. Forshey was superintendent of the Texas Military Institute (Lat. 30° N., Long. 96°25′ W. of Greenwich), in 1858, 1859, and 1860.

Among these observations we find the following, which seem to furnish *consistent* examples under the *Cases* described in (85); and the list might readily be extended.

Case 1st.

Evening of Oct. 5, 1858; 1 day before new moon.
Evening of Nov. 6, 1858; 1 day after new moon.
Evening of Nov. 7, 1858; 2 days after new moon.
Evening of March 3, 1859; ½ day before new moon:—
Light narrow, except near the horizon, and towering high.

Case 2d.

Evening of Oct. 12, and morning of 13, 1858; between new moon and the first quarter. A midnight band of light seems to be delineated; such as will also be noted among the observations under *Case 5th.*

Approaching to the conditions of *Case 2d:*—
Evening of March 31, 1858; 2½ days after full moon.
Evening of Nov. 10, 1858; 3 days before the first quarter of the moon.
Evening of Nov. 13, 1859; 3½ days after full moon.
[The three last-mentioned instances are specially described in *Note* 3 to (72).]
Evening of Nov. 11, 1858:—
This observation may be specially classified with the preceding three. It was made three days before the first quarter of the moon. The position, therefore, is nearly that of *Case 5th.*

Case 3d.

Evening of April 22, 1859, 2 days before the last quarter of the moon.

Figure seems to show the peculiar bright spot indicated in the description of our *Case 3d*, of this *Article.*

Case 4th.

Evening of Oct. 29, 1858; day of last quarter of the moon.
Time 11*h.* to 12*h.* P. M.

[1] Page 217 [2] Page 298.

A midnight band with parallel edges. The figure seems to indicate that the band was about 7° or 8° wide. The appearance is such as it might be if the light were reflected at all but right angles to the girdle.

Evening of April 4, and also that of April 5, 1858; two and one days, respectively, before the last quarter of the moon.

In the evening of April 5, the light is expressly noted as being visible "entirely across the heavens, from Aries at least to Libra."

Evening of Oct. 27, 1858; nearly one day before the last quarter of the moon.

The light seems to have, consistently, been short but considerably bright.

Both characteristics are more distinctly manifest, in the evening of Oct. 28, 1858; day of the last quarter of the moon.

Evening of Dec. 28, 1858; about 2 days after the last quarter of the moon.

Light short and rounded at the top, and the base very broad.

Evening of Jan. 15, 1860; 1 day after the last quarter of the moon.

Light described as having been "intensely bright;" and, in the drawing, it tapers rapidly.

(91) Among the *Notes on the Zodiacal Light*, by Rev. Samuel J. Johnson (*Proceedings of Royal Ast. Society for March*, 1874), we find—"What Humboldt speaks of as the 'mild pyramidally-shaped zodiacal light, very visible to the unassisted eye' has been displayed here" (at *Upton Helions Rectory*, *Crediton*) "this winter with far more distinctness than I have noticed since Feb. 21, 1870, when I witnessed a vivid appearance of the phenomenon from Lytham, on the Lancashire coast. It was conspicuous, amongst other nights, on February 8, when the impression that Tycho mistook the light for the 'abnormal vernal evening twilight,' appeared at first sight almost pardonable."

This seems again to present an example of our *Case 5th.*

"Feb. 16. Sky clear for a brief interval about 8 P. M. The conical figure very fairly defined, except at the apex, where the curvature was somewhat difficult to make out. *Mars*, situated nearly on the axis; about which point the light seemed equal in brightness to that portion of the *Milky Way* that passes through *Cassiopeia.* Near the horizon the intensity was decidedly greater, *ν Ceti* appeared just outside the cone of light; the head of *Aries* faintly involved in it; it could be traced, though with difficulty, 3° or 4° above the *Pleiades.*"

Again, a remarkable example of our *Case 1st.* For this was *the day* of the *New Moon*, and the moon was $1\frac{1}{4}$ day from the *Perigee.* Confirmed this is withal by the next observation.

"Feb. 18. Could be readily followed before the moon set. . . . . Clear extent at the base 30° to 35°. Not quite so brilliant as on the 16th; I fancied a slight reddish tinge in the brighter portions."

Appropriately descriptive of our *Case 2d.*

"March 6. The Zodiacal Light again conspicuous. In extent and general features unaltered; in intensity scarcely so great. The clearest defined portion lay between *ν Ceti* and *γ Arietis;* at lower altitudes the light, although brighter, appeared very much diffused. *Mars* about 5° left of the axis."

An example of our *Case 3d.* "The clearest defined portion" was nearly *opposite* to the moon, then 3½ days past the full, and 1⅓ day beyond the apogee.

"March 7. With regard to the earliest visibility of the light, it was not noticeable till 15m. after stars of the brightness of γ *Arietis* had shone out, and not quite so soon as the *Milky Way* at equal altitudes. Its whiteness more dusky than the latter. At an altitude of about 20°, η and α *Piscium* (the latter just within the boundary) were somewhat dimmed by its intensity."

This is followed by another *Note on the Zodiacal Light*, by E. B. Knobel, Esq., who writes from *Stapenhill Burton on Trent*, and says: "I would beg to direct attention to the unusual brilliancy of the Zodiacal Light this winter. . . ." On two clear evenings in the first week in January, on January 17, at 6.45 P.M., and, lastly, on Feb. 8, at 7 P.M., it appeared as an elongated luminous cone, the apex of which, on January 17, extended nearly to the star γ *Arietis*, and on Feb. 8, the apex just enclosed η *Piscium.*

"It appeared nearly as bright as the *Milky Way*, and sufficiently bright to attract the attention of a casual observer.

"I should mention that my situation is quite away from the town, and sufficiently high to be above the mists of the valley."

The observation of Jan. 17 affords another good example of our *Case 1st;* the date being a little more than ½ a day before *New Moon*, and about 3 days before the moon arrived at the *Perigee.*

The observation of Feb. 8 confirms that of Rev. Samuel J. Johnson of the same date, previously quoted.

These observations are, moreover, all confirmatory of those made about the same time, as well as at other dates, at the College of New Jersey, by the author of this paper; and which, indeed, furnished the data for the distinction of the various *Cases.*

[A very little observation will suffice to make it very evident, that under circumstances in other respects entirely similar, the fact of the atmosphere being *dry* will notably affect the apparent extent as well as brightness of the Zodiacal Light; in accordance with the special, and even uniform, experience of Col. Forshey, already referred to in (72) and *Note.*]

(92) Chaplain Jones also speaks of pulsations in the Zodiacal Light; as having been observed by himself and others. His synopsis of these observations at p. XIII of his Introduction is: "Some time early in 1854 I saw in a newspaper a brief notice of the pulsations of the Zodiacal Light seen at Kew Observatory; but as the newspaper did not state where they were observed, or the authority, and as I had now been observing for a year without having noticed anything of the kind, I set it down as an ocular deception, and the thing passed entirely from my mind. But in March of this year (see No. 111), I was surprised, one evening, at seeing the Zodiacal Light fade sensibly away, dimmed to almost nothing, and then gradually brighten again. This was repeated several times; but the effect, after all, was to leave me only in amazement and doubt; subsequent nights, however, gave abundant exhibitions of this kind, of which, with the times and changes, I have

made ample records with the particularity which the case required. It was a great satisfaction, after my return home, to find that Baron Humboldt had observed the same thing while in southern latitudes, though he thought it more probable that it was owing to 'processes of condensation going on in the uppermost strata of air, by which the transparency, or rather the reflection of light, may be modified in some peculiar and unknown manner.' My records, however, will show that there is a regularity of appearance at the closing off of these pulsations, which proves that they do not belong to so uncertain a cause as atmospheric changes, but to the nebulous substance itself. They seem to intimate a great internal commotion in the nebulous matter, for they were too rapid to be occasioned by irregularities in its exterior surface.

"I noticed them again the following year, but must refer the reader to my records and charts. The changes were a swelling out, laterally and upwards, of the Zodiacal Light, with an increase of brightness in the light itself; then, in a few minutes, the shrinking back of the boundaries, and a dimming of the light; the latter to such a degree as to appear, at times, as if it was quite dying away; and so back and forth for about three-quarters of an hour; and then a change still higher upward toward permanent bounds."

(93) That these pulsations should be real seems not incredible in the instance of a substance having, as it would seem, a density even less than that of the material which exhibits the rapid changes of intensity, etc., of the aurora borealis. The girdle, moreover, would have a very nearly constant position with respect to the earth and the moon—*both magnetic;* and the earth in a relatively rapid *rotation.*[1]

(94) It would seem most probable that the middle plane or equator of the girdle should nearly coincide with the plane of *the moon's* orbit; but even in that case, the more intense illumination by transmitted light would be in directions nearly parallel to the plane of the ecliptic. That, and the local illumination, (75), ascertained and described by Mr. Jones, would together make it difficult to determine where the middle plane may be situated; though some observations of the "gegenschein" might seem to make it the same with the plane of the moon's orbit.

The position of the vertex of the Zodiacal Light would need to be more carefully scrutinized, and compared with that condition.

Such being the state of things, observations for *parallax* must, withal, most probably continue to be unsuccessful.

(95) As a summation of the consistencies of the hypothesis of a nebulous girdle revolving around the earth in the same time and general direction with the moon, and exhibiting the phenomena of the Zodiacal Light, we have:—

1. That it provides a conservative force for the maintenance of such an appendage.

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[1] But it would be more difficult to understand and account for these special phenomena presented by the material in question, if it were directly a solar, instead of a terrestrial, appendage.

2. It will account for the phenomena common to all appearances of the zodiacal light, *broad base* and all.

3. It accounts for certain periodical changes in form and intensity, etc., of the same, which seem to be completed in a synodical revolution of the moon.

4. It provides for the *gegenschein* in form and position; and possibly also for "a lunar zodiacal light."

5. It renders a plausible account of the fading, at times, and total disappearance of the Zodiacal Light.

6. It accounts for the absence of a determinate parallax of the girdle.

7. It shows why, when east and west zodiacal lights are visible at the same time, the middle, even, of the zodiacal arch need not be wholly obscured by the earth's shadow.

8. It provides for the "pulsations."

Origin of the Girdle.

(96) It remains to consider how far the origin of the girdle may be accounted for by the modified nebular hypothesis, already so frequently applied.

If the moon herself were formed of a spheroidal shell [such as those described in (37)], while the form of the earth with its expanded atmosphere was yet very oblate; the equatorial diameter extending beyond the present distance of the moon—*i. e.* more than 60 times the radius of the earth's equator—the moon, derived from the atmosphere of this spheroid, might, at first, indeed have had the form of a spheroidal shell, with its equatorial circle nearly *in the plane of the ecliptic*, as the orbit of the moon now is, instead of *the plane of the earth's equator, since determined.*

This whole collection of material having, by processes heretofore described, (26), been brought to revolve together, the outer portions having thereafter failed to be collected with those that went to form the moon herself, these same outer portions would still continue to revolve and complete the same periodic time.

The part between the moon and the earth would nearly all be compelled to fall toward the earth in obedience to her superior attraction; except, possibly, some small remnant still forming an extra-mundane nebulosity (the middle of it at the position *A* in *Fig.* 13); the existence of which might help to account for some of the phenomena of *solar eclipses*, if not also of those of *transits* of the inferior planets; which it would be out of place to enlarge upon in this connexion.[1]

(97) Whether the material which exhibits the *Aurora Borealis*, or rather *Aurora Polaris*, can have had a similar origin, near to the pole of the oblate expanded atmosphere, and so, also, near to the pole of the Ecliptic in direction, as well as actually near to the earth, can be little better than matter of conjecture. The results, of the spectrum-analysis [(74) and *Note*] do not yet establish a composition

[1] The present Astronomer Royal, Sir George B. Airy, is understood to have said, soon after the total eclipse of the sun, in 1842, that some of the phenomena of that eclipse required for their explanation the supposition of the existence of a material *between* the moon and the earth.

of this material similar to that of the Zodiacal Light. It may, however, be asserted that auroral phenomena are most intense in latitude *about* that of the arctic circle; in which region, it must also be remembered, we have the magnetic poles. It is withal true, that the Zodiacal light seems sometimes to have exhibited (like the Aurora) a *ruddy* tint. An instance is mentioned in (91).

Saturn's Dusky Ring.

(98) The situation of the dusky ring of Saturn somewhat resembles that of the zodiacal girdle (if supposed to be a terrestrial appendage). But the shape of the dusky ring is different from that of the girdle; and its position, concentric with that of Saturn [7 of (43) and *Note*], is maintained by the action of many satellites instead of one; the total action of the several bright rings on particles within being in every case *zero.* But the dusky ring besides is, as it were, *walled in* by the bright rings, which themselves are kept concentric with both the planet and the dusky ring.

Of the Inclination of the Planes of the Orbits of the Planets and Satellites to the Equators of their respective Primaries; and the relative positions of their Perihelia and Nodes.

(99) In a Memoir on the Secular Variations of the Elements of the Orbits of Eight Principal Planets, its author, Mr. John N. Stockwell, M.A., has given us the maximum and minimum inclinations of the planes of those orbits to the invariable plane of the solar system.[1]

From these and the inclination, 7°15′, of the plane of the solar equator to the plane of the ecliptic of 1850, as ascertained by Mr. Carrington,[2] we obtain the following approximate inclinations of the planes of the orbits to the plane of the sun's equator; carrying the reference back to that ancient state of things in which the nodes (of the same name), of the sun's equator and those of the planets' orbits in the invariable plane, respectively *coincided.*

Inclination of Orbit to Sun's Equator.			
With . . .	Minimum Inclination to Inv. Plane.	Mean Inclination to Inv. Plane.	Maximum Inclination to Inv. Plane.
Mercury	0°56′	1°18′	3°31′
Venus	5 40	4 58	2 24
Earth	5 40	4 37	2 34
Mars	5 40	2 42	0 16
Jupiter	5 36	5 28	5 11
Saturn	4 53	4 46	4 39
Uranus	4 45	5 9	4 33
Neptune	5 6	4 59	4 53

[1] *Smithsonian Contributions to Knowledge*, vol. xviii, p. 169 of the Memoir in question.
[2] As quoted in Sir J. Herschel's *Outlines of Astronomy* (11th edition), (392).

It will be observed that when the planes of the orbits most nearly coincide with the invariable plane, they yet make an angle of nearly 5° with the plane of the sun's equator, except in the instance of Mercury, in which the inclination is scarcely 1°; while the Earth and Venus, under the variety of circumstances here indicated, still, as it were, assert their *character as half-planets*, by preserving among themselves always nearly the same inclination.[1]

In view of our hypothesis all along kept in view, the question would here seem to be a pertinent one—Why so great an average deviation in the planes of the planetary orbits from the plane of the sun's equator?

The answer to this may, perhaps, be found in what has heretofore been insisted on; viz. the acquisition of material in the nebulous state from *extra-equatorial* portions of the sun's atmosphere; it being added withal that such an acquisition would not take place from both the northern and southern half-spheroids *at the same time.*[2]

The extra-equatorial acquisition, (37), of more dense material being thus mainly from one side, *that* has, it would seem, tended to produce an average deviation in the plane of the resulting orbit.[3] In that aspect of the matter, and, in view also of the *Ancient State* contemplated in (44) and in Table (F), it may not be entirely without significance that the color of Neptune is a pure white, while that of Uranus is inclined to yellow, and that of Saturn, the other component [as in Table (F)] is decidedly so. But Jupiter is, again, white, while Mars is ruddy, and the Asteroids are—Juno of a pale yellow color, and the others reddish.[4]

Then, again alternately, the half-planet Venus, and also our satellite are both white; while Mercury is nearly of a rose color.[5] In the case here supposed, it is

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[1] With M. Sporer's value of the inclination of the sun's equator, the numbers in column 2*d* will be diminished 18′.

[2] An examination of Mr. Trowbridge's paper, already referred to [*Notes* to (38) and (57) respectively], shows that he has wrought with the same idea in view; though he has applied it to the change in the solar axis of rotation.

[3] Unless, with Mr. Trowbridge, we say that "the invariable plane of the solar system must" (also) "be the invariable plane" for "the primitive solar spheroid, and that it must have coincided approximately with the plane of the sun's equator;" and so he compares the inclination of "the invariable plane" to the ecliptic with that of the orbit of Neptune, with which it nearly agrees. In such a case, with the average existing inclination of the plane of the sun's equator to those of the planetary orbits; it would seem that the sun's equator has itself changed its position; the vicissitudes being similar to those, (68), which, according to M. Laplace, the earth in its forming state seems to have undergone.

But it should here be borne in mind that the invariable plane has its position ascertained by a reference to the conditions of material as *now* accumulated into planets with well-determined orbits; and so the invariable plane thus conditioned may very possibly be not coincident with "the invariable plane of the *primitive* solar spheroid."

[4] *Le Ciel*, par Amédée Guillemin, 4ième Edit. pp. 283 and 284.

[5] Are the white planets, then, in part derived from the one half-spheroid, and the planets of another color from the other? and is the half-spheroid, which furnished the white series, the *northern* one? (?)

For, as respects the existing state of comparative activity in the two hemispheres of the sun, as indicated by the appearance of the solar spots, "a very material difference in their frequency and magnitude subsists in its northern and southern hemisphere; those on the northern preponderating besides manifest that what would be the *ascending* node of the planetary orbit when, in such a case, the acquisition was from the one half-spheroid, would be the *descending* node in the instance of the other.

And with respect to the matter here brought into question, as well as in other aspects, though without deciding that they have any significant connexion; we may consider some of the relations developed by Mr. Stockwell, and exhibited in his *Memoir;* such as—

"The *mean motion of Jupiter's node* on the invariable plane *is exactly equal to that of Saturn, and the mean longitudes of those nodes differ by exactly* 180°."

The latter portion of that description may have some interest in this connexion.

Mr. Stockwell states, withal, that "The mean angular distance between the perihelia of Jupiter and Uranus is exactly 180°."

These and other relations connected with them, are shown by Mr. Stockwell to be eminently harmonious and conservative; and then, after stating that he had prepared separate solutions corresponding to several increments of the Earth's assumed mass; and that a comparison of the values which the different solutions give for the superior eccentricity of the Earth's orbit "has suggested the inquiry whether there may not be some unknown physical relation between the masses and mean distances of the different planets."[1]

After having withal arrived at the conclusion that "a system of bodies moving in very eccentric orbits is" . . . . "one of manifest instability;" he says, "and if it can also be shown that a system of bodies moving in circular orbits is one of unstable equilibrium, it would seem that between the two supposed conditions, a system might exist which should possess a greater degree of stability than either," and then indicates a superlatively grand problem, viz., that "The idea is thus suggested of the existence of a system of bodies in which the masses of the different bodies are so adjusted to their mean distances as to insure to the system a greater degree of permanence than would be possible by any other distribution of masses." He adds: "The mathematical expression of a criterion for such distribution of masses has not yet been fully developed; and the preceding illustrations have been introduced here, more for the purpose of calling the attention of mathematicians and astronomers to this interesting problem than for any certain light we have yet been able to obtain in regard to the solution."[2]

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in both respects" [Sir J. Herschel's *Outlines*, etc., (393)]. See, also, the enumeration and classification of solar spots, founded upon Mr. Carrington's observations, as reported by M. Faye (*Comptes Rendus*, tome lxxvi, p. 393).

The white planets Jupiter and Venus seem to show in their atmospheres, *now*, traces of great activity, even such as ould be consistent with a high temperature. As respects Jupiter, see again *Note* 2 to (69).

[1] See pp. xiv, xvi, and xvii of the Introduction to the *Memoir*, respectively.

As to the existence of such a relation and also as to its connexion with the times of rotation of the several planets—see, again, *last Note* to (44); also *Article* (109), and *Consistency* 61*st* of the *Summation* in (110).

[2] See pp xiv, xvi, and xviii of the Introduction to the *Memoir*.

(100) In the *satellite systems* we find the orbit of the outermost satellite of *Saturn* making an angle of about 14° with the plane of his equator and that of the rings, this angle being about one-half of that which the latter makes with Saturn's orbit, while the orbits of the other satellites are nearly in the plane of rings and the equator.

Then the orbit of *our own moon* has a mean inclination of something less than 5° 9′ to the *orbit* of the Earth; while the variable inclination to the *Earth's equator* is *more than four times as great;* as though the moon in the nebulous state had been "abandoned" in the form of a spheroidal shell *before* the axis of the earth, (68), was *established;* and so with Saturn's outer satellite, under it may be even more disturbing circumstances, (43); while the orbits of the inner satellites and the rings of Saturn, having a *later history*, nearly coincide with the plane of his equator, the same being very nearly the case with the satellites of *Jupiter;* the outer one, notwithstanding, justifying its character as shown in Table (D) in (20), by exhibiting an inclination greater than that of either of the other three.

The orbits of the satellites of *Uranus* are nearly perpendicular to the plane of his orbit; and so that their motions are even retrograde; while the equator of the planet [3 of (43)], inclined at an angle of about 79⅓°, has its rotation direct; all exhibiting, as it would seem, the effect of the great transference of material to Saturn, described in (43).

And although, at present [see 3 of (43)], the equator is inclined to the orbits of the satellites at an angle of about 60°; yet, if it be indeed allowable to refer the situation of all these to that *very* ancient time when the *ascending* node of the equator on the planet's orbit nearly coincided with the *descending* nodes of the orbits of the satellites, then all would be found approximating to a coincidence in the *same plane*, the several inclinations of *all of them* to the plane of the planet's orbit being now near to 79°; but the *direction of rotation* of the planet the *reverse* of that of the *revolutions* of the satellites.

It might almost seem then, as if, in the great *transference of material* to the ancient Saturn here again spoken of, *the rotation of the outer, and mostly rarer, portions of the mass had been most affected; so that, in the satellite-formation, the resultant rotation became even retrograde, while the condensing planet conformed to the usual result of a direct rotation; though (in what was apologetically characterized as the tilting up of this whole system) all were constrained to revolve in planes nearly at right angles to the planet's orbit*, and all nearly in the *same* plane.

The satellite of *Neptune* revolves in an orbit having a large inclination to the plane of the planet's orbit, and the motion is retrograde; but whether that also marks the direction of the rotation of the planet's equator, does not yet appear; nor *which* direction, therefore (that of revolution, or that of rotation), might be regarded as having been established before the other.

The Minor System.

(101) After the separation of the great mass of Jupiter, the "abandonment of the solar atmosphere would seem to have again occurred more exclusively in the region of the solar equator; and thus the Asteroid-mass and Mars appear to have been separated; to be succeeded in order, and with variety of constitution, by the Earth, Venus, and Mercury.

And so it would appear, on a smaller scale (*within more restricted limits* for the balancing of the centripetal and centrifugal forces), was constituted *that minor system*, which, *in fact*, resembles the whole great solar system, in the features and mode of constitution already traced in changes on the larger scale. A system, viz., in which the Asteroids and Mars, as far as may be, have the places respectively of Neptune and Uranus on the greater scale, and the Earth and Venus those of Saturn and Jupiter [the Earth, (39), greater than Venus, from the accession, from regions of the sun's atmosphere other than equatorial]. After these Mercury [and possibly an interior planet], to have the place analogous to that of all the small planets (not Asteroids) in the great solar system.

Resemblances and Differences between Saturn and the Earth.

(102) It may not be without some interest to exhibit in connexion the resemblances and differences between Saturn and the Earth—the Saturn of this Minor System. These are:—

1*st.* In ancient times, an unusual oblateness of form, evinced [(43) and (96)] in the case of both planets by the great distances of their satellites; the outer satellite of Saturn, and also our own moon, being each at the distance of more than 60 radii of its own primary.

2*d.* Saturn and the Earth have each an abnormal density; that of Saturn being *too low*, it would seem, because of the absorption, (43), of the rare material, which would otherwise have constituted the half-planet interior to Uranus; but the Earth's density, (39), being made abnormally *great* by the absorption of an extra-equatorial portion of the sun's nebulous atmosphere.

3*d.* Each of these planets exceeds the other planets in the same region of the solar system with itself, in number of satellites. This is true, though the Earth has but one; but that is the only one in the Minor System.

4*th.* Saturn is surrounded by two systems of bright rings and a dusky ring; and the Earth [if we admit the existence of the Zodiacal Girdle, (78)] is surrounded by something analogous to the dusky ring of the other planet; though they differ from one another to some extent, both in form and position; and the one is preserved because the planet has *many* satellites, the other because its planet has but *one* such accompaniment. [See, again, 7 of (43), and (79) to (83) inclusive.]

5*th.* The Earth [2 of (39)] seems to have been instrumental in producing the great inclination of the equator of its interior half-planet Venus, and Saturn [3 of (43)] as efficient in producing a similar effect upon the half-planet exterior to itself, viz., Uranus.

(103) The analogies to the great planetary system, presented by the satellite systems, have been discussed, in another connexion and aspect, in (58) and (59).

Possible Succession of Changes, in the Progress of the Division, Recombination, and Final Separation of the Great Masses of the Solar System.

(104) In the *Ancient State* contemplated in (44) and in Table (F) in (45), the relation of masses and distances was, it would seem, very nearly the same with that of the existing masses and distances of Jupiter and of Saturn as exhibited in (53); viz., that in which $m(r)^2$ of the one $= m'(r')^2$ of the other.

For—retaining the symbols in (44)—[the second mass in order in Table (F) in (45), including in itself the masses of Uranus and Saturn, while the first mass is that of Neptune]; we have in the instance of the second mass

$$m'(r')^2 \text{ of } [(\mathrm{U})\hat{♄}] = 0.05090861;$$

and for the first,

$$mr^2\psi = 0.0458582;$$

the ratio of the two being

$$\frac{m'(r')^2 \text{ of } [(\mathrm{U})\hat{♄}]}{mr^2\psi} = 1.1101;$$

which, since mr^2, thus, nearly $= m'(r')^2$, gives

$$\frac{m}{m'} = \frac{(r')^2}{r^2};$$

or the masses nearly in the inverse ratio of the squares of the distances.

Next, comparing the mass and distance of Neptune—also those of the whole-planet (U), made up of *Uranus* and its (now) missing *interior half-planet* ♅i—and then, the mass and distance of $\hat{♄}$, that is of Saturn in its *ancient* state before, (43), ♅i was absorbed [the mass of ♅i being deduced as in (41)]; we shall obtain for the several ratios of the distances and the *inverse ratio* of the $\frac{3}{4}$ *powers of the masses*, respectively:

$$\frac{\text{dist. of } \psi}{\text{dist. of } (\mathrm{U})} = 1.7770; \qquad \frac{(m')^{\frac{3}{4}} \text{ of } (\mathrm{U})}{m^{\frac{3}{4}} \text{ of } \psi} = 1.7687.$$

$$\frac{\text{dist. of } (\mathrm{U})}{\text{dist. of } \hat{♄}} = 1.7908; \qquad \frac{(m'')^{\frac{3}{4}} \text{ of } \hat{♄}}{(m')^{\frac{3}{4}} \text{ of } (\mathrm{U})} = 1.7125.^{1}$$

And then, with respect to the existing *Saturn* and *Jupiter*, we have, as in (53),

$$\left.\begin{array}{l} m'''(r''')^2 \text{ of } ♄ = 0.025985 \\ m^{\mathrm{iv}}(r^{\mathrm{iv}})^2 \text{ of } ♃ = 0.025832 \end{array}\right\};$$

a coincidence more perfect than that found in the instance of the two outer great masses, in which the data to be used are less accurately ascertained. Then here,

[1] It is at least curious that Saturn deprived of the mass of ♅i (*i. e.* the *ancient* Saturn) must here once more enter into the computation instead of the existing planet.

of course, again, the masses are very nearly in the inverse ratio of the squares of the distances.[1]

The history of the changes would then seem to be:—

1. That the division of the great masses, *Neptune* and that composed of *Uranus* and *Saturn*, *first* occurred; in accordance with a proportion of masses and powers of distances, such as *Jupiter* and *Saturn* now present.

2. That *afterward* occurred the division of the compound *Uranus-Saturn mass* into the masses of the whole-planet (U) and the *ancient Saturn* ♄.

3. That *subsequently to that*, the material of the whole-planet (U) was rent [the outer half-planet Uranus possibly falling inward somewhat, to justify the new equilibrium of forces];[2] and, (43), the material of the *inner* half-planet ♅*i* passing over and combining with the ancient *Saturn* ♄, to form the mass in part of the existing *Saturn* ♄.

4. That, *before the planetary character of Saturn was complete*, the mass [derived in great part, it may be, from the atmosphere of the other half-spheroid of the sun],[3] which was to form *Jupiter*, became temporarily *blended* with the *Saturn-mass;* to be in the end separated in accordance with the same law of arrangement of masses and distances which, at first, was prevalent in the instance of the great masses, *Neptune* and the combination of *Saturn-Uranus*.[4]

(105) It will be observed, that the preservation of the continued equality of ratios here in question, depends upon the introduction, in one connexion, of the *ancient* Saturn, that is Saturn deprived of the very mass acquired by the process which brought about the disappearance of the mass of the *interior* half-planet ♅*i*, as the same is described in (43) and (44), and the proof of which is manifold; while the preservation of an equality of ratios in another connexion is as truly dependent on the introduction of the *whole* mass of the *existing* Saturn.

Such are the facts; and no explanation appears, except that of the process which bore away the mass of the *interior* half-planet, the reality of which seems thus, again, to be confirmed; to which, possibly, may be added the mode of subsequent combination and separation suggested in (104).

Then we have the negative evidence, that the supposititious separation of the great masses in question in any other way, is not found to yield at all similar proportions.

Kirkwood's Analogy.

(106) This Prof. Daniel Kirkwood communicated to the *American Association* for the Advancement of Science in 1849.[5]

He first speaks of what, (39), we have described as the neutral point.

Thus, as Prof. Kirkwood states it (and the same is applied to the Earth in our

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[1] The *existing* and *not the ancient* Saturn appearing here.

[2] See 5 of (43).

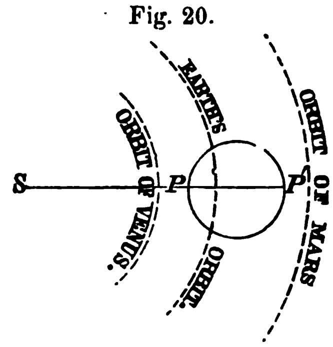
[3] See (99) and *Note*.

[4] In this connexion—see, again, *Articles* (56) and (57).

[5] *Proceedings*, p. 208.

figure): "Let P be the point of equal attraction between any planet and the next interior, the two being in conjunction; P' that between the same and the one next exterior.

Fig. 20.



"Let also $D =$ the sum of the distances of the points PP' from the orbit of the planet" (the whole PP' in the figure); "which I shall call the diameter of the sphere of the planet's attraction.

"$D' =$ the diameter of any other planet's sphere of attraction found in like manner.

"$n =$ the number of sidereal rotations performed by the former during one sidereal revolution round the sun.

"n', the number performed by the latter; then it will be found that

$$n^2 : n'^2 :: D^3 : D'^3; \text{ or } n = n'\left(\frac{D}{D'}\right)^{\frac{3}{2}}\text{"}$$

From this we shall have, alternately,

$$n^2 : D^3 :: n'^2 : D'^3; \; i.e.$$

$$\frac{n^2}{D^3} = \frac{n'^2}{D'^3} = a \; constant.$$

The coincidence with fact is very close in the several instances of *Venus*, the *Earth*, and *Saturn*.

The proportion thus exhibited is *analogous* to Kepler's 3d Law; that the squares of the periodic-times of the planets are as the cubes of their mean distances from the sun; and it is hence called *Kirkwood's Analogy*.

An "Examination" of this by the late Sears C. Walker is also given in the *Proceedings of the American Association* for 1849 (pp. 213 to 219 inclusive), and its consistency with *Laplace's Nebular Hypothesis* made the subject of comment.

Failure of the Analogy in the Case of Uranus.

(107) Conceding that the time of rotation of Uranus [3 of (43)], as found by W. Buffam, Esq., viz. 12 hours $\pm$, is a first approximation to the truth; Kirkwood's Analogy will be found to fail in the case of Uranus.

For if we apply Mr. Walker's formula, in which θ represents the time of rotation (a mean solar day of the Earth being $= 1$); a, a planet's mean distance from the sun; and D, the diameter of the (Kirkwood) sphere of the planet's attraction; then,

$$\theta = \left(\frac{a}{2D}\right)^{\frac{3}{2}};$$

and we shall find, with the values of masses and distances as given in our Table (A), in (3), that, in the instance of Uranus,

$$\theta = 1^d.30380+ = 31.291 \; hours.$$

instead of nearly 12 hours; the result of the observation already quoted.

But even this negative result seems almost like *a shadowing forth of the* catastrophe, which happened when the material of the half-planet interior to *Uranus* [(43) etc.] passed over to Saturn; which has so often asserted itself in our preceding investigations.

With the half-planet restored to its place [its distance as in Table (B), in (14), and its mass, as in (41)], we shall have, by a comparison of Uranus, with that and with Neptune, and the application of the formula,

$$\theta = 31.883 \textit{ hours};$$

agreeing nearly with the former result.[1]

But if we combine Uranus and the restored interior half-planet, in a whole-planet arrangement at the whole-planet limit (U) in Table (B), in (14); we shall have (by a comparison with Neptune and the *ancient* Saturn ♄, and the application of the formula) for the time of rotation of whole-planet (U),

$$\theta = 16.451 \textit{ hours}.$$

Was there, then, in the collection of material adapted to form a whole-planet at limit (U), the origination of a moment of rotation of the remaining half-planet Uranus, which was not all destroyed when the interior half-planet mass passed over to Saturn??

All this is not for a moment to be insisted upon; but there seems to be a possibility that the failure of the *Analogy* in question, may, in this case, be due to these special conditions here also appearing as if in question; as they have been heretofore.

Approximate Result in the Case of Mars.

(108) In the application to the case of Mars, we may make use of the relative asteroid-mass as made out in (46); viz., 0.58929 of the mass of Mars.

Then, as in (60) the indications were in favor of a half-planet arrangement of the asteroid-mass, we have—distributing the mass [*Note* to (51)] in accordance with *that*—the *interior* half-asteroid mass = 0.33745 of the mass of Mars; and the distances withal [in accordance with the *Laws* found in (10)] being derived from those in the region in question (viz., *Saturn* to *Mars* inclusive), as exhibited in (12).

From these and the masses, on the one side, and the mass and distance of the Earth on the other, we may then obtain D, the diameter of Mars's sphere of attraction; and then, Mr. Walker's formula,

$$\theta = \left(\frac{a}{2D}\right)^{\frac{3}{2}},$$

will give for Mars's time of rotation $27h.\ 34m.8$.[2] Observation gives $24h.\ 37m.4$. The coincidence is as close as could be expected; the masses being more or less uncertain, and the formula confessedly "approximate."

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[1] For the interior half-planet ♅i, if it ever had the planetary form and state, the time of rotation would be $33h.982$.

[2] Deriving the distances from the more extended series in the column of Law in Table (B), in (14), we have $27h.46m.3$, for the time of rotation.

[With a *whole-planet* arrangement of the asteroid-mass, the resulting time of rotation of Mars would be 19*h*.968; the half-planet arrangement of (60), thus appearing again as preferable.]

So that, in the case of the asteroids, although the component material has been dispersed; yet, as a half-planet portion has not passed over and been absorbed by an interior planet, the determining conditions of the next interior planet's rotation have, it would seem, not been entirely disturbed.

Of "Bode's Law," and the reasons for its success in the approximate determination of the respective distances of Uranus and several other planets, and also for its failure to determine the distance either of Saturn or that of Neptune.

(109) The most simple statement of the (so-called) Law of Bode (or of Titius) is that of Sir J. Herschel; viz.: . . . . "The interval between the orbits of the Earth and Mercury is nearly twice that between those of Venus and Mercury; that between the orbits of Mars and Mercury nearly twice that between the Earth and Mercury; and so on."[1]

Now, (13), the mean value of our whole-planet ratio is (stated here approximately) 1.8. But, if we subtract Mercury's distance from each of two successive terms in the whole-planet series, to obtain the *intervals* between orbits here in question, the ratio of the remaining intervals will exceed the ratio r of 1.8+, since the smaller of the two distances compared will be *more than proportionally* diminished by such a subtraction; and the value of greater divided by the less (*i. e.* here of the ratio) will be *increased*. Thus:—

$$\frac{\textit{Asteroid limit (A)}}{\textit{Mars' distance}} = 1.8+$$

But

$$\frac{\text{(A)}\textit{—Mercury's distance}}{\textit{Mars'—Mercury's distance}} = 2+;$$

the ratio being a very little greater than that which "Bode's Law" requires.

The same ratio is, even, very well justified in the instance of the Earth compared with Venus, and Mars with the Earth; though [as exhibited in Table (B) in (14)], while the ratio of the distance of Venus to that of Mercury is (incidentally) the whole-planet ratio r, that of the Earth's distance to that of Venus is only $r^{\frac{1}{2}}$, and even the ratio of Mars' distance to that of the Earth is only $r^{\frac{3}{4}}$. But the increase of the measuring unit in the comparison, as we proceed, and the subtraction of Mercury's distance in every instance (one being more effective in the one case, and the other, in the other) together make the one interval near to the *double* of the other.

The ratio, as has been already stated, nearly accurate for the Asteroid-*interval* in the middle of the whole-planet series. But, when we pass beyond that to the Jupiter and Saturn terms, successively, the subtraction of only Mercury's distance, though just about sufficient for the justification of the Jupiter interval, gives a result *too small* in the instance of that of Saturn.

[1] *Outlines of Astronomy* (11th Edition), (505)

Thus—making use of the veritable distances as stated in Table (B), expressed approximately, we shall find:—

$$\frac{\textit{Jupiter's distance} - \textit{Mercury's distance}}{\textit{Asteroid distance} - \textit{Mercury's distance}} = \frac{4.81}{2.43} = \frac{1.98}{1}.$$

But

$$\frac{\textit{Saturn's distance} - \textit{Mercury's distance}}{\textit{Jupiter's distance} - \textit{Mercury's distance}} = \frac{9.16}{4.81} = \frac{1.90}{1}.^{1}$$

The same process would fail notoriously in the case of the next whole-planet (U), were that yet to be found. But Uranus being an *exterior* half-planet, the ratio of its distance to that of Saturn is $r^{\frac{5}{4}}$ instead of r; and so the double interval for Uranus is tolerably well preserved in comparison with that of Saturn.

But as the ratio of Neptune's distance to that of the *exterior* half-planet Uranus (though on a larger scale than that immediately preceding, in the order here pursued) is only $r^{\frac{3}{4}}$, the subtraction of only Mercury's distance from each of the others, leaves the interval for the greater in a ratio to that for the less of not more than $\frac{1.52+}{1}$; and so, the representative number when it ought to be 301 appears in the series of numbers illustrating the "law" as 388.

The latest application of "Bode's Law" would seem to be that of Maxwell Hall, Esq.; an abstract of whose communication is given in the *Monthly Notices of the Royal Astronomical Society*, vol. XXXIV, No. 7 (May, 1874), under the title of "*The Solar and Planetary Systems.*"

The author states "Bode's Law" as follows: "In the solar and planetary systems the mean distances of the planets do not greatly differ in value from the terms of the series:

$$4\lambda,\ 7\lambda,\ 10\lambda,\ 16\lambda,\ 28\lambda,\ 52\lambda,\ 100\lambda,\ 196\lambda,\ 388\lambda,\ \text{etc.},$$

where λ has different values in different systems. But there may be more than one, or there may be no planet or satellite near any of the above theoretical distances."[2] And he then proceeds to determine λ in miles for the planetary system, and for the Jovian, Saturnian, and Uranian satellite-systems respectively.

"Some of the numerical coincidences are very close; thus in the Uranian system, taking the distances to be 7λ, 10λ, 16λ, and 28λ, the first three satellites give $\lambda = 17600$, and 17100, and 17600 miles respectively (but the fourth satellite gives $\lambda = 13400$ miles)."[3]

"He then states a second proposition: 'Twice the unit of length in any system

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[1] Accordingly in the statement of the "Law" as not unfrequently made, which represents the successive distances by the numbers 4, $4+1\times3$, $4+2\times3$, $4+2^2\times3$, etc., Saturn's representative number exhibits a conspicuous failure. For instead of the true number 95, the distance is represented by 100; the veritable distance—as has, in effect, been stated—being too *small* to conform to "Bode's Law."

[The representative numbers 4, 7, 10, etc., appear in Mr. Hall's series, quoted in this *Article.*]

[2] Especially in this connexion, see *Note* to (7).

[3] What has already been stated in the way of exposition of the application of this (so-called) law in the planetary system, and an inspection of our Table (E) in (21), with its *two* ratios in accordance with veritable laws, will at once show the reason for this discrepancy. See also *Note* to (7).

is approximately equal to that distance which corresponds to the period of rotation of the central body of that system,' or say"

$$\lambda = 1580 M^{\frac{1}{3}} P^{\frac{2}{3}},$$

where $M =$ mass of central body, in terms of the mass of the earth, P the period of the axial rotation in hours, λ in miles as before.

It thus appears that dividing the value of λ for any system by the value of $M^{\frac{1}{3}} P^{\frac{2}{3}}$ for the central body of the system, the quotient should be 1580. For the Solar, Jovian, and Saturnian the quotients are 1790, 1340, 1720, mean 1620. For the Earth $\lambda = 13100$; so that regarding the Moon as a fourth satellite (the three interior ones missing) the theoretical distance is 210,000 miles.[1]

The paper concludes with some considerations as to M. Lescarbault's planet *Vulcan.*

[Sir J. Herschel, in a *Note* to Article (505) of the 11th edition of his *Outlines of Astronomy*, makes the following statement:—

"Another law has been proposed (in a letter to the writer, dated March 1, 1869), by Mr. J. Jones, of Brynhyfryd, Wrexham. If the planets' mean distances from the sun be arranged in the following orders: Mercury, Venus, Jupiter, Saturn; the Earth, Mars, Uranus, Neptune; the product of the means in each group is nearly equal to the product of the extremes.

$\frac{Venus \times Jupiter}{Mercury \times Saturn} = \frac{Earth \times Neptune}{Mars \times Uranus} = 1.$ In point of fact the first fraction $= 1.02$, and the last $= \frac{1}{1.03}$, so that the approach to verification of the law is really very near."

Now the first fraction

$$\frac{Venus \times Jupiter}{Mercury \times Saturn},$$

may be resolved into

$$\frac{Venus}{Mercury} \times \frac{Jupiter}{Saturn}.$$

An inspection of the ratios exhibited in our Table (B), in (14), will show that the first of these component fractions expresses a whole planet ratio r; and the second component the *inversion* of that, $\frac{1}{r}$. So that the value of the whole expression

$\frac{Venus \times Jupiter}{Mercury \times Saturn}$, resolved into its two components here specified $= \frac{r}{1} \times \frac{1}{r} = 1.$

Then the other fraction, $\frac{Earth \times Neptune}{Mars \times Uranus}$, may be resolved into $\frac{Earth}{Mars} \times \frac{Neptune}{Uranus}$;

[1] The error is here nearly $\frac{1}{8}$ of the quantity to be determined; whereas in our Tables (B) to (E), and even (F), inclusive, the greatest difference between veritable Law and Fact is that in the instance of Uranus, in which the discrepancy is not $\frac{1}{60}$ of the quantity to be measured, and even for that [5 of (43)] a special reason is assigned. In almost every other instance the discrepancy is far less than that; indeed, all but incomparably small. The greater differences specified in Mr. Hall's paper are such as are characteristic of "Bode's Law."

and, from Table (B) again, we learn that the first of these component fractions expresses the *inversion* of an *exterior* half-planet ratio $\frac{1}{r^{\frac{3}{4}}}$, and that the second component expresses the *exterior* half-planet ratio $r^{\frac{3}{4}}$ itself. So we have the value of $\frac{Earth \times Neptune}{Mars \times Uranus}$ resolved into $\frac{1}{r^{\frac{3}{4}}} \times \frac{r^{\frac{3}{4}}}{1} =$, again, to 1.

The small differences from 1 (in the one way and the other) in the actual values already quoted, are due to the slight increase in the value of the ratio r (and its derivatives); as exhibited in our *Article* (13).

For the arrangement, otherwise, into the two "orders" here first quoted, there is no very manifest reason; and so it would seem to be merely artificial.][1]

Summation of Coincidences.

(110) In the summation of coincidences and the comparison of the same with theoretical deductions, those will be first considered which have at various times been indicated by commentators on the nebular hypothesis of Laplace, beginning with those which M. Laplace has himself specified, and of which his hypothesis was especially designed to furnish the explanation.

1*st.* The motion of the planets in the same circular direction, and nearly in the same plane.

2*d.* The motions of the satellites, with few exceptions, in the same direction with those of the planets.

3*d.* The rotation of these different bodies and of the sun, also in that same circular direction, and in planes not much inclined to one another.

4*th.* The small eccentricity of the orbits of the planets.

5*th.* The hypothesis accounts for the existence of comets in the solar system, as well as the variety of inclination of their orbits; also for the very great eccentricity, and the change in the form of the same. *See* (34), and Note VII of the *Système du Monde.*

[M. Laplace's expansion and explanation of these five coincidences is exhibited in our *Articles* (24) to (34) inclusive.]

6*th.* The hypothesis accounts for Saturn's rings, (28), and that they also revolve in the same circular direction with the planets and their satellites.

7*th* Asteroids as well as ordinary planets are provided for; as is explained in (29).

8*th.* The great heat of the sun and, possibly also, of some of the existing planets, are facts in place.

[1] Though it is also *curious* that we have, in both the instances in question, the product of the expressions of *white* planet distances, divided by that of those which are not of that description; the reason for the classification of the planets in that respect even, having (99), at least a quasi-relation to the *Ancient State* of the system exhibited in Table (F), in (45); which is again related (in the connexion in question) to the *more recent* arrangements exhibited in Table (B), in (14).

[See in this connexion (69) and its *Note*[2]. The seeming perturbations of the atmospheres of *Venus* and *Mercury*, and even those of the atmosphere of Jupiter, are also consistent with the supposition of a high temperature.]

9*th*. The very existence of a gaseous or nebulous envelope of the sun, as well as of the atmospheres to so many of the planets, is itself consistent with the hypothesis in question. [Confirmed by recent investigations with the spectroscope].

10*th*. Another evidence of previous high temperature, as the hypothesis would require, is found in the internal heat of the Earth, even now.

11*th*. Similar is the evidence of geological facts; many of which require the existence of a very high temperature in ancient times.

12*th*. The evidences of the effects of a former high temperature in the moon, supplement the evidence of geology.

13*th*. The hypothesis accounts for the lack of an atmosphere to the moon; in the explanation quoted in (69).

14*th*. The hypothesis, in like manner, accounts for the absence of secondary satellites (satellites of satellites); and also shows why there are no secondary rings; in the explanation quoted in (68).

15*th*. The hypothesis accounts for the arrangement by which the moon and (it may be) the other satellites, present the same faces severally to their respective primaries; the explanation being that quoted in (68).

16*th*. The hypothesis accounts for the *spheroidal form* of the planets; they having been supposed to have been, in older times, in a gaseous or in a liquid state, in which they took a form suited to the rotation of their gravitating material. The researches of Prof. H. Hennesey "have shown that the ultimate ellipticity" in consequence of the accumulation of water, etc., in the equatorial regions, and the gradual abrasion of polar continents in case the Earth were *at first* a *solid* sphere, would be $\frac{1}{404}$, instead of "that found by actual measurement;"[1] viz, a little greater than $\frac{1}{300}$. The *Earth* could not then have been solid at first. The oblateness of *Mars* seems to be too great; but it is supposed that the liquid surface of some planets was solidified before they could assume the figure appertaining to their rotation.

17*th*. The molecular constitution and whole composition of *aerolites;* so like, and yet in some respects so different from, what we find on the earth, is consistent with a common origin of all from the ancient solar atmosphere. [The spectrum-analysis has, within a recent period, afforded similar testimony, and to a greatly enlarged extent].

[The existence of the Zodiacal Light is also consistent with the hypothesis in question. This consistency is not numbered here; as it must appear in another connexion.]

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[1] As stated by Prof. Kirkwood —*American Journal of Science and the Arts*, for Sept. 1860, p. 167.

18th. We have Kirkwood's Analogy; already discussed [(106) to (108) inclusive].

19th. It is consistent with Laplace's Nebular Hypothesis that the large planets should be furnished with satellites, while the small planets are not so attended, with the bare exception of the Earth; which, even, has but one, unless some small bodies, not wholly unlike aerolites, are to be added to the number. The "abandonment" of nebulous rings, etc., could more readily proceed and be carried to the result of condensed rings, or of satellites, in the case of the larger bodies.

20th. The greater density of the smaller planets in comparison with the larger; and the tendency to a law of increase from without inward, in the whole series; as manifested in Table (A) in (3). The decidedly abnormal deviations from this are specially accounted for. [See references in exposition of Consistencies *32d* and *39th* respectively.]

21st. The Nebular Hypothesis furnishes M. Laplace with an explanation of the exact commensurability of the angular motions, and thus of the periodic times, etc., of Jupiter's satellites; they having "immediately after their formation not moved in a perfect vacuum." The action, in this case, of a resisting medium, itself consistent with his hypothesis, is illustrated by M. Laplace in the way already indicated in (67).

The farther summation of consistencies will have special reference to other phenomena and relations discussed in this paper.

22d. In addition to Consistency *21st*, we have an approximate commensurability of periodic times of some of the *satellites* of Saturn, and also of those of *the four outer planets* of the *Solar System;* as detailed in (67).

23d. The modification of the Laplace Nebular Hypothesis, (37), providing for spheroidal shells, provides, also, for a conservative force for the holding together of great masses; and so prevents the indefinite multiplication of asteroids in all regions of the system.

24th. As if in consistency with a common origin and mode of development, we have the three laws of distances of planets and half-planets, as stated in (10); and the arrangement in accordance with these, in Table (B), in (14).

25th. We have also the prevalence of similar laws in the *System of Saturn;* the arrangement in accordance with which is exhibited in Table (C) in (18 Then, moreover, we have the arrangement in so far as a more restricted system would admit (viz., in accordance with two such laws) in the *System of Jupiter;* as shown in Table (D) in (20); and in the approximate arrangement of the *System of Uranus* in Table (E) in (21).

26th. The gradual and systematic increase or diminution, as the case may be, of the leading ratio, and its powers in these several systems, would seem again to indicate that the arrangement had a *physical* origin, not unlike that under discussion. [See the Summing up of these relations in (22).]

27th. The consistency of the results obtained in so many connexions by a reference of positions to the *centres of gyration* of the revolving masses, together with other facts in the same connexion, all but *insist upon and require* that the masses in question must have turned around *together*. [See especially the application of this in (39) and (41); also (44) with Table (F) in (45); and (53), (54), (56), and (104).]

28th. The conditions involved in connexion with what is stated in Consistency *27th*, also show that the law or laws of apportionment of the masses are not independent of the laws of the distances; but that they are *functions*, one sort of the other. [See, again, last *Note* to (44); also quotations in (99), and *its last Note but one.*]

29th. It is in perfect agreement with Consistency *26th* and *27th*, if not also with Consistency *28th*, that the *rings* of Saturn referred to their respective *centres of gyration* have, in Table (C), in (18), the places of *satellites.*

30th. We have, besides, the commensurability of the periodic times of the two great satellites of Saturn with those due to some of the limits of Table (C) in (18), at which satellites are now missing, as that commensurability is exhibited in (66), and in consequence of which (in view of the Laplace Hypothesis, or of that hypothesis as modified) the existence of satellites may have been prevented there; and thus also possibly may have been occasioned the space between the two systems of Saturn's bright rings; all, as explained in (64).

31st. Again we have the commensurability of the periodic time of *Jupiter*, and some of the periodic times due to certain of the *asteroid limits*, and also that of *Mars;* which may have been the means of breaking up former planets or asteroids, as is also explained in (64). With respect to the special relations of the *half-planets*, Earth and Venus—in accordance with the Laplace Nebular Hypothesis, or else with the same modified as in (37), we have:—

32d. The abnormal density of the Earth accounted for (a density too great for the Earth's place in the system). [See 1 of (39).]

33d. In connexion with that, we have the great inclination of the equator of the other half-planet Venus to the plane of its orbit; apparently accounted for in 2 of (39).

34th. We have the approximate agreement of the neutral point (the Kirkwood limit of the Earth's sphere of attraction between the two half-planets on that side) with the *whole-planet* limit for the combination of the two masses; as exhibited in 4 of (39). [The approximation to an agreement also of this last with the centre of gyration of the two half-planets has already been adverted to in the exposition of Consistency *27th*, and its reference.]

35th. The great *oblateness* of the nebulous Earth (with its accumulated dense material) is, (96), recorded in the great distance of the moon, = to full sixty equatorial radii of its primary planet.

36th. That the ascertained *density* of the moon should be but 0.55654 of that of the Earth is another fact in place in this discussion, in view of Consistency 35th.

In *consistency with the rest, and in confirmation of our subsidiary hypothesis accounting for the disappearance of the now-missing half-planet, which should be* *found interior to Uranus; viz., that its mass was absorbed by what previously constituted the mass of Saturn,* we have:—

37*th*. That the mutual attractive force of the missing mass and the *then-existing* Saturn was adequate in measure to the effect supposed; as is explained in 1 of (43).

38*th*. That the limit to which the same mutual attraction extended is itself not very far short of the limit (U) at which the *whole-planet* mass would be likely to be rent; as in the Earth-Venus case [4 of (39)]; as is farther explained in 2 of (43).

[The mass of the missing planet is found in (41) by the application of the formula for the centre of gyration; which has its reference in Consistency 27*th*.]

39*th*. The very *inferior density* of Saturn [below that due to his place in the system, and the least in all the series of densities of planets in Table (A) in (3)], is here a special fact in place; so much of the material of the existing Saturn being derived from the region *outside*. [See 4 of (43).]

40*th*. All this would contribute to give the forming nebulous Saturn a *very oblate* figure; the ellipticity being even greater than that of the forming Earth —for the outer satellite Japetus is at the distance of more than sixty-three radii of its primary; and the very faint light of that satellite in certain positions may be accepted as one condition not in itself inconsistent with a low density.

41*st*. All this would permit the formation of *satellites* to begin and advance, some time before that of the rings; and so the *conservative* influence of the satellites be exerted, in those early times, to preserve those rings and keep them concentric with the shrinking planet; and thus make it possible for Saturn to be adorned with those remarkable appendages which make him an *instantia solitaris* in the system. [See explanations and quotations in 7 of (43) and its *Note* 3.]

42*d*. The great mass of the *ancient Saturn* ♄, (notwithstanding its low density), would seem to have been efficient in bringing about the great inclination of the equator of Uranus to the plane of its orbit, as well as to that of the ecliptic, [and also that of the whole Uranian system, specially described in 3 of (43);] the whole so like the effect on the inclination of the equator of Venus, insisted on in Consistency 33*d*. Thus these two phenomena, so like, but which present themselves in regions of the system remote from one another, are found to be referable to the action of not unlike causes.

43*d*. The very considerable inclination of the Saturnian system (equator of the planet, rings, and orbits of satellites)—so unlike in that respect to the system of the other great planet Jupiter—would seem itself to be referable to the same disturbance which so *tilted up* the equator and all the system of Uranus.

44*th*. It is not inconsistent with all this, that on a comparison of the column of Fact with the column of Law in Table (B) in (14), Uranus would almost seem to have *perceptibly* fallen in; and Saturn perhaps have been drawn a little outward. [See 5 and 10 of (43)]. And it may be that Consistency 31*st* is also to be found here [see 9 of (43)].

45*th.* A like effect may be more distinctly traced in the system of *Saturn*, in the instance of the satellite Hyperion, which is just outside of Titan, the Jupiter of the system; as may be made apparent by a comparison of the columns of Fact and of Law in Table (C) in (18); which is withal explained in (66). That *Mars* also seems to have *perceptibly* fallen in by the acquisition of material from the asteroid mass is discussed in (65).

46*th.* The subsidiary hypothesis of the transference of the half-planet mass, is still farther and very remarkably confirmed by the ratios due to the Ancient State exhibited in Table (F) in (45), the Uranus-Saturn ratio of which is *not justified*, unless we also *restore* Saturn to its *ancient state*, by restoring also the missing planet to its legitimate place; and then combine that, the mass of Uranus, and also that of the *ancient* Saturn ♄̂, all at *their* common centre of gyration; and *then* the appropriate ratio in Table (F) *is very scrupulously justified.*[1]

47*th.* The conformity of the ratios of the *Ancient State* is itself a justification of the *mass* of the missing half-planet; that mass being independently determined in conformity to the condition, that the centre of gyration of that half-planet and Uranus should be the same with the whole-planet limit (U) in Table (B) in (14).

This value of the mass is still farther confirmed, in so far as may be, by the curious relations developed in (104); in which the mass of the *ancient* Saturn ♄̂ (Saturn deprived of the mass of the now-missing planet) enters in one connexion, and the mass of the existing Saturn in another.

48*th.* The justification of the ratios of the *Ancient State*, as the same are exhibited in Table (F) in (45), itself demands a special value of the asteroid-mass; and the value thus ascertained, with the data which we have, agrees closely with that signified by M. Le Verrier (in one of his investigations of the subject), as being required by the *perturbations of the planet Mars.* [See explanations and quotations in (47) and *Note.*]

49*th.* The arrangements of the *Ancient State* exhibited in Table (F) in (45), into which combinations of planetary masses alternately enter, justify the position of Mercury in *their own series.* Then withal the aphelion of Mercury's orbit has a whole-planet place in Table (B) in (14), while the perihelion of the same has a half-planet place. The arrangements of both tables thus consistently indicate that Mercury has accumulated in itself the material appropriate for a planet and a half planet, and that its position justifies that.

50*th.* The arrangements now specified, also serve to account for the *great eccentricity* of Mercury's orbit; the planet having absorbed into itself the ring-like or shell-like masses, one due to the whole-planet position at the aphelion of the orbit, and the other to the half-planet position at the perihelion.

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[1] As the annual aberration of the sun, planets, and fixed stars is without explanation, if we do not admit the doctrine of the earth's motion; but the whole explanation is adequate in mode and in measure with that motion first admitted. There is certainly an approximation to a parallelism here.

51*st.* The distribution of masses which Consistency 50*th* would indicate, and the Laws of Distance in (10), together enable us to compute the mass and mean distance of material (possibly planetary) immediately interior to Mercury. And the mass thus indicated seems to be adequate to produce the perturbations of Mercury's orbit to the *extent required by M. Le Verrier.* [See discussion of all in (52)].

52*d.* With the arrangement of distances of *Jupiter* and *Saturn* either in the column of Law or in the column of Fact, in Table (B), in (14), and with the ascertained value of their masses, we find, (53), the *vis viva* or moment of (simultaneous) rotation of the one very accurately equal to that of the other; so that the masses are inversely as the squares of the radii of gyration; *i. e.* here inversely as the squares of the mean distances from the sun.

There is, at least, a rude approximation to the same, on a large scale, when the masses and distances of Neptune and the next term of the series [U♄] in Table (F) in (45) are, in like manner, made the subjects of a proportion in (104).

It may be then that the great divisions of the nebulous solar atmosphere (antecedent perhaps to other planet-forming developments) were made in conformity to the proportion here in question.

But in what seems like the subsequent subdivision of the [U♄] mass, in its special comparison with Neptune, the proportion, (104), of *distances inversely* as the $\frac{3}{4}$ *power* of the masses is very accurately justified; in which the whole-planet mass (U) (consisting of the mass of Uranus and that due to its now-missing interior ♅*i*) enter, as well as the ancient Saturn ♄; though, as already intimated in Consistency 47*th*, the *existing* Saturn enters in the comparison with Jupiter.

The moments of (simultaneous) rotation of the outer and inner systems of bright rings of Saturn exhibit, (53), an approximation to equality like that of the great outer masses here spoken of.

[Also if the expressions of the respective velocities of the existing ring systems, at their centres of gyration be made to enter, instead of the 2*d* powers of the same, we have, (53), with m and m' for the masses, and a and a' for the distances from the centre of the planet

$$\frac{m \times a \text{ of } inner\ rings}{m' \times a' \text{ of } outer\ rings} = 1.0752.$$

Incidental very possibly, but curious.]

53*d.* From what is stated in Consistency 52*d*, it would seem to have been the case, that the large masses of the system, in the series from without inward, increased in a more rapid ratio than the respective distances diminished (in a more rapid ratio, viz., than the inverse ratio of the distances); the increased density of material more than counterbalancing the effect of its diminished quantity.

Accordingly, in (57), with scarcely an exception, we find a continual increase of the masses, from Neptune to Jupiter inclusive; the mass of Jupiter being transcendently the greatest of all.

The like, (58), is true (Hyperion being the exception there) in the system of Saturn; Titan being the Jupiter of the system; as is, (59), the 3*d* satellite among the four satellites of Jupiter; while, lastly, the Earth and Venus, (101), are, respectively, the Saturn and the Jupiter of the Minor System of planets; and there are other curious relations, furnishing subjects for comparison, which are detailed in (101) and (102).

54*th*. It is shown in (16) that the centre of gyration of a thin homogeneous ring is in the circumference of a circle concentric with the edges of the ring, and bisecting its area. Also that R' and r' being the radii of the edges of the ring and C that of the centre of gyration, we shall have

$$C^2 = \tfrac{1}{2}(R'^2 + r'^2).$$

(*a*) The same, in (54), is extended to the case in which the equivalent masses are both thin homogeneous rings, one wholly clasping the other; R' and r' representing the respective radii of the centres of gyration of the two clasping rings, and C that of the common centre of gyration.

(*b*) The common formula for the centre of gyration will, when reduced, give us the same equation, in the case of *any* two *equal* masses, irrespective of the form of either.

Now although the two systems of bright rings of Saturn can scarcely be presumed to be homogeneous, and although they do not seem to be equal in mass, yet, (55), the equation in question is found to be very nearly applicable to them.

[Making use of this *inductively*, as some indication of the ring-like form in revolving masses, (55), we found, that the like equation in the solar system was very nearly justified in the case of the half-planets Earth and Venus; and, (56), that a similar one was nearly realized in the case of Neptune and Uranus; the distances being those in the column of Law, in Table (B) of (14).[1]

These results might seem to be consistent with the supposition that the flowing over of the material of the oblate solar atmosphere had given to the masses in question, at some period of their development, a form not unlike that of *a thick ring;* and yet the same cannot be regarded as decisive; and in the case of Uranus and Neptune, there is the other explanation found in (*b*) of this Consistency; for the masses of Neptune and Uranus are nearly *equal*.]

In another and different instance we have a closer agreement.

The centre of gyration, (19), of the whole system of Saturn's Bright Rings is at a distance from the planet's centre = 1.9090; being just within the outer edge of the Inner Bright Ring (or Rings), which is at the distance 1.9276; as though the division of one great ring had taken place there.

Some reason why the opening between the system of rings should be permanent, is given in (64); which reason has already been alluded to in Consistency 30*th*.

[1] Before Uranus (Consistency 44*th*) had perceptibly *fallen in*.

55th. An application of the criterion of the ring-like form as stated in Consistency *54th*, was, as far as might·be, made use of [(60), (61), and (62)] in determining as to whether it would be preferable to attribute to the *asteroid-mass* (in the progress of its development) at any period, a whole-planet or a half-planet arrangement; without the assertion that either is, beyond controversy, supposable.

In favor of the supposition of a *half-planet* arrangement, we had:—

(*a*) That we do not find the equation here in question justified when a comparison is instituted between the whole-planet arrangement and Mars; but, with an appropriate distribution of the mass for a *half-planet* arrangement we find, (60), a close approximation to the fulfilment of the equation in question.

(*b*) This might seem to have the less weight, were it not also true that the limit of equal attraction between the exterior half-asteroid mass and Jupiter, (60), is 3.35790, and that between the interior half-asteroid mass and Mars, is 2.14438; which limits very well mark the *range* of the mean distances of the known asteroids; and, (61), the respective distances 3.34083 and 2.47748 of the exterior and interior half-asteroid masses approximate to the *aphelion* and *perihelion distances* of several of the existing asteroids; so that the case in that respect may possibly resemble that of Mercury, commented on in (50).

(*c*) Other circumstances discussed in (65), and referred to in Consistencies 31*st* and 45*th*, seem to indicate that (with the wide range and great eccentricity of the asteroid-orbits) Mars may have acquired material of slower motion; which caused that planet (*perceptibly*) to fall in. Such is the look, when Fact and Law in Table (B) in (14) are compared.

[This is again alluded to here because of its present connexion with the other considerations; though formally noticed in Consistency 45*th*.]

(*d*) Though we may not attribute too much weight to our results when the data are imperfect—yet, in this connexion, we find that the formula derived from Kirkwood's Analogy, which, (107), signally fails (for reasons assigned) to give us the length of the sidereal day of Uranus, yet, (108), approximates to a true result in the case of Mars, referred on the one side to the Earth and on the other to the interior *half-asteroid* mass.

56th. In view of the secular variations of the planetary orbits, we have exhibited in (99) the close approximation to coincidence of the planes of those orbits in very ancient times.

In (99) we make the suggestion that the mean inclination of the sun's equator (of nearly 5°) to these may have arisen from the fact that the acquisition of material of a planet from the extra-equatorial regions of the sun's nebulous atmosphere, may have been mainly from one side; the changes in the two half-spheroids not being *simultaneous*.

But this is a region for speculation in which our sources of information are very restricted. [Not quite discordant with it, however, is the fact mentioned in (99), and *its Note* (5), that the great planetary masses of Table (F) [in (45)] are alternately white and yellow or ruddy.]

57*th*. Other harmonies may be gathered from the *Memoir on the Secular Variations of the Elements of the Orbits of Eight Principal Planets*, by John N. Stockwell, M.A., from which the positions of the planes of the planetary orbits, alluded to in Consistency 56*th*, are taken; which harmonies are to some extent described in (99). These, like Consistencies 22*d* and 31*st*, seem to indicate a common origin of the bodies concerned—under restricted circumstances.

58*th*. As stated in (100), the orbits of the outermost satellites of Saturn and Jupiter have very considerable inclinations to the equators of their respective primaries; as though their development had an earlier history than that of the other satellites and appendages.

And the orbit of *our own moon* has a mean inclination of something less than 5°9′ to the *orbit* of the Earth; while the variable inclination of the Earth's *equator is more than four times as great;* as though the moon in the nebulous state had been separated *in the form of a spheroidal shell*, *before* the axis of the Earth was *established.*

The like, withal, would seem, (100), to have happened in the instance of the satellites of Uranus and their primary planet: with additional varieties, themselves, as it were, confirmatory of the supposition of the rending away and absorption by Saturn of the *mass due* to the (now missing) half-planet, which was once connected with *that* of Uranus.

59*th*. In our explanation of the appearances of certain of Jupiter's satellites as dark spots, while they were in transit across the disk of their primary; the conclusion was arrived at, (69), that the phenomena were due to absorption, and possible interference, of the light proceeding from Jupiter and encountering that of the satellite; as is explained in (69). The circumstances also seemed to indicate:

(*a*) A confirmation of the supposition that the satellites, in their revolution, continue to present, respectively, each nearly the same face to its primary.

(*b*) That the phenomena of absorption, etc., indicate, as a reasonable probability, that the satellites are *colder* than their primary.

(*c*) That, therefore, the satellites, like our moon, have very possibly little or no atmosphere.

(*d*) That, in view of the Laplace Nebular Hypothesis, the satellites may, then, possibly have lost their atmospheres, in the same way in which M. Laplace supposes the moon's atmosphere may have been carried away; which was already alluded to in Consistency 13*th*, and explained in (69).

All this bears upon the question of a similar origin and development of all the bodies (comets excepted) of the solar system.

60*th*. In Articles (70) to (95) inclusive we have a discussion of the phenomena of the *Zodiacal Light;* which, in (78), are regarded (in modification of Chaplain George Jones's hypothesis) as due to *a girdle* encompassing the Earth. It is further indicated, in (79), that the girdle is preserved from destruction by having its periodic time coincident with that of the moon; and the limits of the girdle, (82), are computed in accordance with that subsidiary hypothesis, and the variations, (83), in the size of the girdle are distinctly stated. Also *tidal* actions at the ends of the major diameter. Accumulations of material, or the contrary, must also exist, in the maintenance of the dynamical equilibrium *where* the central forces of earth and moon act at an angle with one another; somewhat, it may be, like that which appears in *Fig.* 14, at *Article* (80).

Examples of observed phenomena are afterwards given; and in (95) eight particulars are specified, in which the whole hypothesis seems, thus far, to be consistent with the observed phenomena.

The resemblances and differences of the Girdle and Saturn's *Dusky* Ring are stated in (98).

61*st*. The late Sears C. Walker in a personal communication to the author of this paper, made some years since, was understood to say, that he had computed what would be the time of rotation of the now existing Earth, if its material were given a ring-like form extending to the Kirkwood limits; and that he had found a year for the time of rotation, as the Laplace Nebular Hypothesis would require.

Prof. Benjamin Peirce, commenting on the explanation of the rotation of the planets on their axes, as deduced from the nebular hypothesis of Laplace, and reasoning especially with regard to Jupiter and Saturn, is understood to have "demonstrated, by a mathematical analysis of the movements of the particles constituting the liquid ring, that the velocities of the resulting rotations of those planets must be such as are actually observed." No authentic information of this, however, seems as yet to have been made public.

[Then Maxwell Hall, Esq., (109), would establish a connexion between the mass of a central body, sun or planet, and its period of axial rotation, and certain approximate ratios developed from the so-called Bode's Law.]

In the statement of Consistencies no allusion has been made to the coincidences in the times of revolution of the planets with the respective times of rotation of the sun with an atmosphere supposed to be expanded successively to the distances of the planets. Sufficient data for this are not attainable.

Other coincidences not sufficiently accurate have not been insisted on in the enumeration; and conjectures, like that in (97), with respect to the *Aurora*, cannot yet be verified. The giving of undue weight to the result, in any instance, has, withal, been carefully guarded against.

In view, however, of all the consistencies which have now been enumerated, the inquiry whether these *can all be incidental*, would seem at once to suggest its own negative answer.

But whether that, indeed, be so or no, a single additional statement should, if possible, once for all, be made emphatic:—

THE SPECIAL RELATIONS EXHIBITED IN SECTION II. (DESIGNEDLY STATED WITHOUT REFERENCE TO ANY THEORETICAL CONSIDERATIONS), AND THE OTHER PHENOMENA DETAILED IN SECTION III., AT LEAST IN SO FAR AS MERE NUMERICAL RELATIONS ARE CONCERNED—ALL THESE, FROM FIRST TO LAST, DEPEND UPON EXISTING FACTS OR RELATIONS IN THE SOLAR SYSTEM ITSELF; AND SO MUST ENDURE WHILE THE SYSTEM LASTS, THOUGH EVERY HYPOTHESIS WITH REGARD TO THOSE RELATIONS SHOULD BE REJECTED.

But if every hypothesis be rejected, the relations exist as more or less consistent, but yet as ultimate facts; *i.e.* without any explanation; while the hypothesis, or rather theory, which has been discussed in these pages, seems, with a more or less perfect applicability, to *include and grasp the whole.*

ADDENDUM.

Consistency 62*d*. In addition to what is already stated as a part of Consistency 55*th*, it may be noted, that the resulting rotation of Mars as determined by Kirkwood's Analogy, (108), is not merely, *in so far as may be*, confirmatory of the half-planet arrangement of the *asteroid-mass* exhibited in (60); but also of the *value* of the *mass itself*, as determined in (46): the appropriate fraction of the mass entering into the computation of the time of rotation in question.

NOTE (A).

On the Origin of Clusters and Nebulæ.

The application of similar principles to those involved in the Nebular Hypothesis of Laplace, but on a larger scale, and with reference to a greater variety of circumstances, led the author of this paper to his own hypothesis of the *Spheroidal Origin of Clusters and Nebulæ;* which represents those groups and conglomerations as being the derivations of spheroids (or of rings derived from spheroids, or of masses of an ancient ring-like form) all rotating in a state of dynamical equilibrium, at periods very remote. But, that the process of cooling brought about like phenomena to those which the Laplace-hypothesis maintains to have taken place in the instance of our sun; viz. the same more rapid rotation, sometimes with a local increase of actual velocity, sometimes with a diminution of the same; but always, on the whole, with an increase of angular velocity, continued, however, until the *centrifugal force of rotation o'ermastered cohesion and gravitation*, and, in place of an "abandoned" equatorial ring, portions of the ruptured material were *ejected;* to be left behind the others, in the direction *opposite* to that of the rotation—the material thus being broken into elongated fragments, and they again into drops; but every drop having in it material sufficient to form a condensed nebula, or in the end a star: the result presenting appearances such as are visible in the very beautiful nebula H. 1173; the spirals described and figured by the late Lord Rosse; the projections from the one end of the annular nebula in Lyra; and the teeth *leaning backward* in the globular cluster H. 1968, etc. etc.

The expositions in the communication here referred to, occupy in all twenty-nine (double-column) quarto pages of the 2d volume of (*Gould's*) *Astronomical Journal*, published in 1852; and among those expositions is one, drawn out in detail, the heading of which is "*The Milky Way—a Spiral;*" which is found in No. 37 of the Journal specified, at p. 101; followed by some reasons for supposing that the spiral had four branches, and a dense central cluster.[1]

For a variety of other details as well as a more complete exposition of the phenomena and their progress, reference must be made to the memoir itself; but one of its concluding paragraphs should, if possible, be made emphatic; and, therefore, we also introduce it here. It reads thus:—

"While it is even to be expected that errors may hereafter be found in the various details which have been so fully exhibited, it is respectfully submitted whether this same hypothesis of the spheroidal origin of so many of the clusters and nebulæ, in its most important features, is not adequate *in mode;* or whether, in the very least, the phenomena do not even *require* the admission of *a dynamical equilibrium destroyed*, as the one pervading principle—guiding, as it would also seem, to the explanation of all the other conditions."

It would seem, indeed, to be in vain to look for an exposition of the phenomena and their progress, if we do not keep in view and adhere to the hypothesis of a dynamical equilibrium *destroyed;* a *conservative* view does not *now* suit the case.

Among the conditions requiring just *that*, are the phenomena here briefly adverted to; and the fact that the centres of clusters do not exhibit the enormous condensation *anywhere*, which the "clustering power" of Sir William Herschel, it would seem, must *somewhere* have produced; but, on the contrary, the central portions uniformly appear as if, when they were released from superincumbent pressure, by the rupture of the outer portions of the spheroid, or other primitive form, their feeble central attraction could no longer preserve them in form; and so the centres are always broken up. The *sudden curvature of the spirals*, moreover, seems to be more like that due to the ejection of material under the influence of an excess of centrifugal force, than that which would result from a rushing inward, in obedience to an excess of attraction.

The supposition of original *nebulous* spheroids does not seem to be contradicted by the revelations of the *spectroscope;* but, on the contrary, to be consistent with them.

In further justification of an hypothesis, the distinguishing feature of which is the utter destruction, on the *large* scale, of a dynamical equilibrium, we also reproduce the conclusion of the communication already referred to, which is as follows:—

The more condensed clusters (other things being equal) must, upon this plan, be regarded as probably of the more recent origin; instead of being the older, as supposed by Sir William Herschel (*Phil. Trans.* for 1789, pp. 224 and 225); and if a continued dispersion is even yet in progress, the *permitted* collisions regarded

[1] This assuredly must have been overlooked, or else—though noticed—have been forgotten; or we would not find among the *Proceedings of the Royal Astronomical Society* (Dec. 1869), "A *New* Theory of the Milky Way, by R. A Proctor, B. A.;" which describes and figures the Milky Way as being *a spiral*—though not, indeed, with four branches.

by Sir John Herschel [*Outlines of Astronomy* (872)] as quite supposable as consequences of the clustering power, will be the more frequently avoided; and stars, which, like our sun, may have planets in their keeping, will bear their attendants away beyond the reach of harm.

In view, then, of even the little that has yet been ascertained, may we not in all humility ask whether *this* was not indeed *the way* in which the SUPREME DISPOSER of both great and small events executed his vast purposes; the changes being, alternately, destructive and conservative.

For the growing leaf is fed by the exhalations which it finds in the atmosphere; and the leaf, in its decay, nourishes the vegetating tree; the roots of that tree are embedded in the *débris* of a comparatively ancient earth; the earth itself, in view of the nebular hypothesis (of Laplace), has been detached from the sun; and the sun and other stars would now seem to be but the comparatively small fragments or drops of greater masses: the one great plan pervading the *whole*, being, **BY MEANS OF A PERMITTED DESTRUCTION, TO PROVIDE FOR A MORE PERFECT ADAPTATION AND DEVELOPMENT.**

NOTE (B).

Of the Nebular Hypothesis of Sir William Herschel.

On this subject, Sir John Herschel says in his *Outlines of Astronomy*, (871):—"The first impression which Halley, and other early discoverers of nebulous objects received from their peculiar aspect, so different from the keen, concentrated light of mere stars, was that of a phosphorescent vapour like the matter of a comet's tail, or a gaseous and (so to speak) elementary form of luminous sidereal matter. Admitting the existence of such a medium, dispersed in some cases irregularly through vast regions in space, in others confined to narrower and more definite limits, Sir W. Herschel was led to speculate on its gradual subsidence and condensation by the effect of its own gravity, into more or less regular spherical, or spheroidal forms, denser (as they must in that case be) towards the center. Assuming that in the progress of this subsidence, local centers of condensation, subordinate to the general tendency, would not be wanting, he conceived that in this way solid nuclei might arise, whose local gravitation still further condensing, and so absorbing the nebulous matter, each in its immediate neighborhood, might ultimately become stars, and the whole nebula finally take on the state of a cluster of stars. Among the multitude of nebulæ revealed by his telescopes, every stage of this process might be considered as displayed to our eyes, and in every modification of form to which the general principle might be conceived to apply. The more or less advanced state of a nebula towards its segregation into discrete stars, and of these stars themselves towards a denser state of aggregation round a central nucleus, would thus be, in some sort, an indication of age. Neither is there any variety of aspect which nebulæ offer, which stands at all in contradiction to this view. Even though we should feel ourselves compelled to reject the idea of a gaseous or vaporous 'nebulous matter,' it loses little or none of its force." [The spectroscope indicates that *that* need not always be.] "Subsidence, and the central aggregation consequent on subsidence, may go on quite as well among a multitude of discrete bodies under the influence of mutual attraction, and feeble or partially opposing projectile motions, as among the particles of a gaseous fluid."

"(872) The '*nebular hypothesis*,' as it has been termed, and the *theory of sidereal aggregation* stand, in fact, quite independent of each other, the one as a physical conception of processes which may yet, for aught we know, have formed part of that mysterious chain of causes and effects antecedent to the existence of separate self-luminous solid bodies; the other as an application of dynamical principles to cases of a very complicated nature no doubt, but in which the possibility or impossibility, at least, of certain general results may be determined on perfectly legitimate principles."

"Among a crowd of solid bodies of whatever size, animated by independent and partially opposing influences, motions opposite to each other *must* produce collision, destruction of velocity, and subsidence or near approach towards the center of preponderant attraction; while those which conspire or remain outstanding after such conflicts, *must* ultimately give rise to circulation of a permanent character. Whatever we may think of such collisions as events, there is nothing in this conception contrary to sound mechanical principles."

"Ages which to us may well appear indefinite may easily be conceived to pass without a single instance of collision, in the nature of a catastrophe. Such may have gradually become rarer as the system has emerged from what must be considered its chaotic state, till at length, in the fulness of time, and, under the pre-arranging guidance of that DESIGN which pervades universal nature, each individual may have taken up such a course as to annul the possibility of further destructive interference."

To which we may add, that it is well understood, that, with respect to *all* this, Sir J. Herschel has but fully and clearly expressed the very thoughts and feelings of his distinguished father.

[The supposed "aggregation," in view of what is stated in *Note* (A), must be regarded as being a wider segregation, by the continuance of an even now progressive *dispersion*.]

In so far as the *nebular hypothesis* here under consideration, has, at least, the character of an ingenious conjecture in the form of a generalization, it would seem to relate to a more ancient state of things than that contemplated in our *Note* (A); being indicative of the way in which the rotating spheroids there described *might* themselves have been formed.

The existing phenomena seem to *require* the spheroids to have preceded the present state of things; but there is very little to indicate what must have been the state of the material composing the spheroids before *they* acquired their form.

The revelations by the spectroscope of a similarity of molecular constitution in so very many instances are not indeed inconsistent with the supposition of a common origin; yet *they* do not *require* that.

The statement of Sir J. Herschel, already quoted, speaks of the "chain of causes and effects" here in question as being antecedent to the existence of self-luminous solid bodies.

Being thus antecedent, the traces of the phenomena which have *required* the admission of such causes and effects have, it would seem, been so far obliterated, in the course of the changes which have since taken place, that the nebular hypothesis here in question cannot now be proved; and yet enough has even here been stated, to show that it cannot be disproved.